Climate Risk in Dynamic Spatial Equilibrium*

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Abstract

We study the economic implications, over time and space, of the aggregate and local risks arising from climate change. We develop a dynamic spatial model of the U.S. economy disaggregated at the county level that features costly forward-looking migration and capital investment decisions, aggregate risk due to uncertain warming trajectories, and local risk due to uncertain exposure to extreme weather. We achieve tractability by leveraging the 'Master Equation' representation of the economy. We estimate structural elasticities and county-level damage functions by matching reduced-form estimates linking population, income, and investment to coastal storms and heat waves. We compare two market structures: island capitalists who hold only local capital, and diversified capitalists who hold capital everywhere. Three lessons emerge. Local risk from shifting weather distributions is far larger than aggregate risk from uncertain warming paths in any given year, although the persistence of aggregate risk makes their lifetime costs comparable for exposed investors. Equilibrium responses of capital prices and diversification insure investors against most of this risk. As a result, the welfare cost of climate risk is small relative to the cost of the warming trend itself.

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1 Introduction

Risk is a central feature of weather and climate patterns. Even with all the advances in weather and climate forecasting, there is substantial uncertainty about how many storms will hit Miami next year or about how extreme and frequent heat waves will be in Houston. Climate change compounds these risks due to geoscientific and policy uncertainties inherent to global long-term temperature scenarios. The consequences of the local and aggregate risk embedded in the evolution of the world's climate are a central feature of the impact of climate change at the local, country-wide, and global levels. In this paper, we study the economic implications of climate risk in the United States over time and across its more than 3,000 counties.

Faced with climate risk, households and businesses will not wait passively for it to resolve, but instead take preemptive actions to adapt as best they can. By moving out of high-risk locations and redeploying capital in low-risk locations, workers and investors can limit their exposure to climate change risk, albeit at a cost. How does climate change risk reshape economic activity when workers and capital can reallocate across space? What is the welfare cost of climate risk after accounting for these forms of costly adaptation?

In this paper, we answer these questions by developing a dynamic spatial model of the United States economy with local and aggregate climate risk. We use a fine spatial resolution at the 3,037-county level to capture climate change impacts and adaptation. Our framework incorporates costly forward-looking migration, capital investment decisions, and multiple dimensions of risk: local realizations of damaging extreme weather and aggregate warming paths are both uncertain. The analysis remains computationally tractable despite aggregate risk in warming trajectories and the high-dimensional state space by leveraging the 'Master Equation' representation of the economy. The ability to compute dynamic spatial equilibria rapidly allows us to estimate structural elasticities and damage functions by matching reduced-form estimates linking population, income, and investment to coastal storms and heat waves.

Our starting point is a framework with forward-looking dynamic migration and investment decisions at a high level of spatial resolution. As in Caliendo et al. (2019), forward-looking risk-averse workers decide where to live and work. They earn an equilibrium wage in their location, which they use to consume and lease housing at equilibrium rents. They receive random preference shocks for locations that motivate them to migrate, subject to bilateral migration costs. Capitalists face a dynamic consumption and investment decision with convex adjustment costs. We consider two market structures that bound their ability to diversify. In the first market structure, which we call the 'diversified model' and generalizes Bilal and Rossi-Hansberg (2025), a repre-

sentative risk-averse capitalist holds capital in every location and trades a risk-free bond in zero net supply. Local capital is then priced with a national stochastic discount factor and commands local risk premia above the national risk-free rate. Investors thus face a spatial portfolio problem based on the risks and returns of capital in all possible locations. In the second market structure, which we call the 'island model' and extends the formulation in Kleinman et al. (2023) to add risk, risk-averse capitalists own the capital stock of the location where they live, must self-finance investment, and cannot trade bonds with each other.

In both cases, capitalists rent out their capital stock to developers who use it to produce housing and commercial structures. Local firms produce consumption goods using labor and commercial structures. The evolution of the local paths of capital depreciation rates, productivities, and amenities depends on climate change with heterogeneous local exposures: as global mean temperatures rise, the frequency of storms and heat waves changes over time, which affects counties heterogeneously.

We introduce aggregate and local risk in this environment. The path of global mean temperature is not known with certainty. Instead, it is subject to persistent aggregate shocks that reflect slow learning about long-run warming trajectories. Thus, local capital depreciation rates, productivities, and amenities fluctuate in tandem with aggregate shocks. Local exposures to capital depreciation, productivity, and amenities are themselves risky. This local risk reflects realizations of rare weather extremes that are specific to each location and possibly spatially correlated. Local risk in capital depreciation and productivity generates local risk premia relative to an economy in which local capital is a safe asset.

The resulting environment is a heterogeneous agent framework with a high-dimensional state space (productivity, population, and capital in every county), rich interdependencies between decisions and prices in all locations, and aggregate risk. To make progress, we leverage the 'Master Equation' methodology from Bilal (2023). Specifically, we use the 'Second-order Approximation to the Master Equation' (SAME), which consists of an analytic second-order perturbation around a long-run deterministic steady state. It allows us to solve efficiently for quantitative general equilibrium counterfactuals in heterogeneous agent models and capture nonlinearities and the effects of local and aggregate risk, by solving a simple set of matrix and three-dimensional tensor Sylvester equations.

Equipped with our solution method, we quantify our framework. In our baseline exercise, we set capitalist risk aversion to a standard value and set worker risk aversion to a lower value since the model abstracts from workers' ability to smooth consumption over time. We verify that our

conclusions are robust to a range of plausible values.

Next, we estimate internally the key parameters that control how agents respond to climate shocks. These are the migration elasticity of workers, the investment elasticity of capitalists, and the damage functions that map global temperature changes into local changes in capital depreciation rates, productivity, and amenities. We use the storm and heat wave probability data for each county collected in Bilal and Rossi-Hansberg (2025). Specifically, we measure how their frequency changes with global mean temperature in each county. We also obtain reduced-form estimates of the effects of these weather extremes on population, income, and investment from Bilal and Rossi-Hansberg (2025), with the associated standard errors. We obtain expert elicitations of the distribution of global mean temperature paths for the next century from the Intergovernmental Panel on Climate Change (IPCC).

We target these reduced-form impulse response functions to quantify our framework. We jointly estimate the migration and investment elasticities, along with the magnitude of shocks corresponding to a single extreme event realization. Given a guess for the two elasticities and a standard calibration of the other parameters, we invert the model in steady state. The inversion delivers all the local characteristics, including local productivities, amenities, and the level of local capital investment costs. To identify the migration and investment elasticities in an outer loop, we target the relative impulse response functions of employment and income per capita to storms and to heat waves, and of investment and employment to storms. To identify damages associated with a single event, we target the level of the overall magnitude of these impulse response functions. We use the computational efficiency of our solution method to propose global identification guarantees.

Storms and heat waves have large effects on capital depreciation, productivity, and amenities. Through the lens of our model, storms cause a 26% capital depreciation shock. Heat waves generate a productivity loss of 2.4%, and, to match the data, require a simultaneous decline in amenities of 0.9%. The resulting damage functions imply that the most exposed counties of the Gulf and southeast Atlantic coasts should expect up to 5.5 percentage points (p.p.) of additional annual capital depreciation per degree of warming. The warmest counties in Florida should expect productivity losses of up to 8.5% per degree.

The last step requires us to convert our estimates of the impact of a single extreme event into damage functions that map global mean temperature to local productivity, amenity, and capital depreciation losses. We interact damages from a given event with changes in the probability distribution of that event for each location in response to changes in global mean temperature.

Combining these objects allows us to construct the effects of a changing climate on the level and volatility of each location's fundamentals.

First, expected changes in capital depreciation, productivity and amenity are identified by the evolution of the first moment—the average—of the distribution of extreme weather. Second, the changes in local volatility of capital depreciation, productivity and amenity shocks are identified by the change in the second moment—the variance—of the distribution of extreme weather. Given that both first and second moments follow from changes in the same binomial climate shock distributions with fixed event severities, variance exposures are proportional to the absolute value of mean exposures across locations. Third, we construct exposures to aggregate risk by calibrating the process of aggregate temperature shocks so that its 95% probability band coincides with the IPCC's warming range by 2100.

We use the quantified model to evaluate the cost of climate risk in the U.S. economy, both in a climate that is 2°C warmer and along the transition to it between 2025 and 2100. Three lessons emerge from these counterfactuals.

The first lesson is that local risk is substantially larger than aggregate risk in any given year. Weather distributions shift rapidly as global mean temperature rises, leading to rising idiosyncratic storm and heat wave risk. The resulting increase in the variance of local damages is much larger than the uncertainty around the warming trajectory itself given the IPCC's likely scenarios.

We gauge the magnitude of local and aggregate risk using the island model as a yardstick to measure how large risk would be if not diversified across locations. In the island model, capitalists on the Gulf and southeast Atlantic coasts require risk premia of up to 2.7 p.p. to hold capital exposed to local risk, against essentially zero for aggregate risk. However, risk premia only measure contemporaneous correlations. As aggregate risk reflects persistent uncertainty about warming trajectories, while idiosyncratic risk reflects annual idiosyncratic variation in extreme event realizations, the present value welfare costs of aggregate risk for capitalists can reach almost three quarters of their welfare cost of local risk in exposed locations.

The second lesson is that despite this rapidly growing risk in exposed locations, the economy appears to be fairly well insured against it. Two forces contribute to this result. First, because investment is costly to reverse at the margin, the price of installed capital rises when a storm destroys the local stock. This equilibrium capital price movement hedges its owners. Without the covariance between the price of capital and local shocks, in the island model the risk premium would rise from 0.6 p.p. to 1.5 p.p. on average on the coast. The lifetime cost of risk for coastal capitalists would rise from 3 to 24% of consumption. The second force is diversification. In the

diversified model, capitalists' cost of risk is only a tenth of a percent since they can diversify their portfolio by investing in locations that do not face this risk, and might benefit from the resulting migration and investment flows. Diversification slightly hurts workers, because capital leaves locations hit by storms faster than workers do.

The third lesson is thus that climate risk is less costly than the deterministic trend in climate change. By 2100, the trend lowers the capital stock by 7% and worker welfare by about 3.5% of lifetime consumption in both models. Risk adds at most half a percentage point to these losses. In the diversified model, capitalist welfare stands at -3.8%, to which risk adds 0.1 p.p. Even without diversification, which leads to an upper bound on the cost of risk, risk represents at most 21% of capitalist welfare losses in 2100.

We explore the robustness of these results to alternative specifications of our framework. First, we find that our conclusions remain similar with Epstein and Zin (1989) preferences that separate risk aversion from intertemporal substitution. Second, in contrast to the baseline model where all idiosyncratic risk is diversified within counties, we introduce uninsurable risk within counties by allowing dispersed damages across capitalists. While our data do not allow us to calibrate the degree of within-location uninsurable risk, we find that plausible degrees of within-location dispersion can increase the cost of risk for capitalists in the absence of diversification.

The methodology that allows us to reach these conclusions is related to the literature on mean-field games and its uses in economics (Cardaliaguet et al., 2019; Achdou et al., 2022). Together, the elements of our framework constitute a heterogeneous agent environment with aggregate shocks, in which the full distribution of workers and capital across counties matters for the economy's evolution. Standard solution methods are not well-suited for the dimensionality and the multitude of equilibrium prices of this framework. To make progress, we leverage the 'Master Equation' representation of the economy (Bilal, 2023), itself formalized by Cardaliaguet et al. (2019) in the mathematics literature studying mean-field games: the state-space representation of our framework in which the distribution is a state variable. We take analytic Second-order Approximations of the Master Equation in aggregate temperature shocks and in the underlying distribution to obtain low-dimensional Bellman equations for the directional derivatives of individual value functions with respect to the underlying distribution and aggregate shocks.[1]

[1]Sequence-space first-order perturbation approaches as in Auclert et al. (2021) and Bilal and Goyal (2025) are not well-suited for our purposes: they stop at first order and thus abstract from risk. In addition, it can be a challenge to even construct and store the Jacobian in a spatial framework with complex interdependencies. A parallel and nascent literature uses neural networks to propose global solutions (Sun, 2025). While this approach sidesteps perturbations, it necessarily relies on functional form extrapolation and lacks convergence guarantees given the challenges associated with training the network on an infinite-dimensional state space.

Our paper is also related to the literature on dynamic spatial models (Desmet and Rossi-Hansberg, 2014; Desmet et al., 2018; Caliendo et al., 2019; Kleinman et al., 2023; Kleinman et al., 2026), an active part of which focuses on climate change (Desmet and Rossi-Hansberg, 2015; Desmet et al., 2021; Cruz and Rossi-Hansberg, 2024; Conte et al., 2026; Cruz, 2021; Balboni, 2025; Bilal and Rossi-Hansberg, 2025). Relative to these papers that focus on perfect foresight transitions or first-order perturbations that feature certainty equivalence, our methodology allows us to introduce aggregate and local risk and compute counterfactuals at a high level of spatial resolution. A related and novel feature of our analysis is that investors face a spatial portfolio problem based on the risks and returns of capital in all possible locations. Furthermore, our computation method also lets us estimate the two key elasticities, the climate damage functions, and local and aggregate risk exposures, by precisely matching the model results with empirical reduced-form local responses. The methodological contribution of our paper is, therefore, to present a dynamic spatial general equilibrium model with high spatial resolution that can be used to study the local and aggregate implications of different forms of risk.

Of course, the main substantive contribution of our paper is to evaluate the economic losses from climate change risk at the aggregate and local levels, accounting for the economy's ability to adapt by reallocating activity across space. An active literature assesses the role of climate change risk in representative agent models or at low spatial resolution (Weitzman, 2009; Weitzman, 2014; Cai and Lontzek, 2019; Barnett et al., 2020; van den Bremer and van der Ploeg, 2021; Olijslagers and van Wijnbergen, 2024; Fried, 2022). By contrast, our methodology enables us to introduce a fine level of spatial disaggregation into our framework, thereby accounting for adaptation and connecting our damage functions and risk exposure to the data. The spatial structure ties the risk exposure of each location to the observed variability of its weather. Sun (2025) is perhaps the only paper that shares our focus on risk in a spatial economy, but does not study the relative importance of aggregate and local risk.

The rest of this paper is organized as follows. Section 2 outlines the framework. Section 3 describes our solution method. Section 4 details our quantification. Section 5 presents our results. Section 6 concludes.

2 A Dynamic Spatial Framework with Local and Aggregate Risk

This section presents the setup of our dynamic spatial model with aggregate and local risk. We model an economy with many locations and two types of agents: workers and capitalists. Work-

ers can move subject to bilateral mobility costs and earn a wage, which they consume each period. Capitalists earn returns on their risky capital investments and face a dynamic consumption-investment decision. We consider two market structures: a 'diversified model' in which capitalists can hold capital in every location and trade bonds, and an 'island model' in which capitalists can only invest in their own location.

2.1 Agents and Preferences

There is a unit mass of infinitely-lived workers who choose in which location $i \in \{1, \ldots, \mathcal{I}\}$ to live. Workers have Constant Relative Risk Aversion (CRRA) preferences $u(C_{it}) = \frac{C_{it}^{1-\gamma}-1}{1-\gamma}$ where $C_{it} = A_{it}\left(\frac{c_{it}}{1-\beta}\right)^{1-\beta}\left(\frac{h_{it}}{\beta}\right)^{\beta}$. C_{it} denotes a Cobb-Douglas aggregator of amenities in the worker's current residential location, A_{it}, a freely traded final good, c_{it}, used as the numeraire, and housing, h_{it}. β denotes the share of housing in expenditure. Workers discount the future at rate ρ. Workers can move at Poisson rate μ, in which case they draw extreme-value distributed idiosyncratic preference shocks for potential destinations, with dispersion parameter ν. If they move, they pay a bilateral moving cost τ_{ij}. A_{it} is determined by the long-run (A_i) and time-varying (a_{it}) components of amenities in location i as $A_{it} = A_i e^{a_{it}}$.

Capitalists have CRRA preferences $\overline{u}(C) = \frac{C^{1-\overline{\gamma}}-1}{1-\overline{\gamma}}$ over the final good, with risk aversion $\overline{\gamma}$ possibly different from that of workers, and do not consume housing. They discount the future at the same rate ρ. We detail the market structures they face below.

2.2 Technology

Workers in location i are employed at wage w_{it} by a representative firm that produces the final good using commercial structures and labor and faces constant returns to scale. Locations are endowed with a local productivity Z_{it} and a local stock of land L_i. The representative firm is endowed with a production function for the final good given by $Y_{it} = Z_{it} S_{it}^{\alpha} (N_{it}^P)^{1-\alpha}$, where S_{it} denotes local commercial structures and N_{it}^P the local number of workers employed in final good production. We define the long-run (Z_i) and time-varying (χ_{it}) components of productivity as $Z_{it} \equiv Z_i e^{\chi_{it}}$. Productivity follows

$$d\chi_{it} = \left(X_{it} - \kappa \chi_{it}\right) dt + \Sigma_{\chi,it}\, dW_{\chi,it}, \tag{1}$$

where $dW_{\chi,it}$ are Brownian motions, independent over time but correlated across locations with $\mathbb{E}_t[dW_{\chi,it} dW_{\chi,jt}] = \mathfrak{c}_{\chi,ij} dt$, so that the covariance matrix of productivity innovations $\Sigma_{\chi,t}$ has entries

$\Sigma_{\chi,ijt} = \Sigma_{\chi,it}\Sigma_{\chi,jt}\mathfrak{c}_{\chi,ij}$. The drift X_{it} and the volatility $\Sigma_{\chi,it}$ depend on climate change, as we specify below. κ controls how fast productivity shocks mean-revert. Since local productivity follows a persistent process, it is a state variable. In our quantification, heat waves determine both productivity and amenity losses, implying that amenity deviations are proportional to productivity deviations: $a_{it} = \eta\chi_{it}$. For expositional simplicity, we impose this condition from now on although it is immaterial for our methodology.

In every location, there is an equilibrium capital stock K_{it}. Capital can be combined with land L_i and construction labor N_{it}^B to produce buildings B_{it} according to the production function $B_{it} = L_i^{\omega}(N_{it}^B)^{\varpi}K_{it}^{1-\omega-\varpi}$. Buildings B_{it} can be costlessly repurposed between residential housing and commercial structures, so that $B_{it} = S_{it} + H_{it}$. Similarly, the total number of workers in location i at time t, N_{it}, can move costlessly between the production and construction sectors, so $N_{it} = N_{it}^P + N_{it}^B$. Denote by r_{it} the common rental rate of buildings to firms and workers. Land rents are aggregated into a national portfolio and rebated to workers in proportion to their wages. A worker earning w_{it} has income $(1+s)\,w_{it}$, where s is the ratio of land rents to wages and is the same in every location and period, as we show in Appendix B.1.

Installed capital is physically immobile across locations. We denote by R_{it} the rental rate of a unit of capital. The local capital stock depreciates by $d\overline{\Delta}_{it}$ over a time interval of length dt, where

$$d\overline{\Delta}_{it} = \Delta_{it}\,dt + \Sigma_{\Delta,it}\,dW_{\Delta,it}. \tag{2}$$

The Brownian motions $dW_{\Delta,it}$ are independent over time and of the productivity shocks, and correlated across locations with $\mathbb{E}_t[dW_{\Delta,it}dW_{\Delta,jt}] = \mathfrak{c}_{\Delta,ij}dt$, so that the covariance matrix $\Sigma_{\Delta,t}$ has entries $\Sigma_{\Delta,ijt} = \Sigma_{\Delta,it}\Sigma_{\Delta,jt}\mathfrak{c}_{\Delta,ij}$. The mean depreciation rate Δ_{it} and the volatility $\Sigma_{\Delta,it}$ depend on climate change, as we specify below. Our Brownian motion specification is general enough to encompass the capital depreciation process that arises when infrequent storms occur at some hazard rate and destroy part of the capital stock. We detail this microfoundation in Appendix A, as it guides our estimation strategy. Investing I_{it} at rate $g_{it} \equiv I_{it}/K_{it}$ requires paying the convex adjustment cost given by

$$c_i(g) = \frac{c_{i0}^{-1/\zeta}}{1+1/\zeta}\,g^{1+1/\zeta} \tag{3}$$

per unit of capital, where ζ is the investment elasticity and c_{i0} a location-specific shifter. The capital stock then evolves as $dK_{it} = (g_{it} - \Delta_{it})K_{it}\,dt - \Sigma_{\Delta,it}K_{it}\,dW_{\Delta,it}$. If we let $\mathcal{K}_{it} \equiv \log K_{it}$, then, by Itô's

lemma,

$$d\mathcal{K}_{it} = \Big(g_{it} - \Delta_{it} - \frac{\Sigma^2_{\Delta,it}}{2}\Big)dt - \Sigma_{\Delta,it}\, dW_{\Delta,it}. \tag{4}$$

We use log capital rather than its level as it enhances the numerical accuracy of our solution method.

2.3 Climate Change

We take the path of global mean temperatures, $\overline{T}_t$, as exogenous. Going forward, most carbon emissions are expected to come from countries other than the U.S. and are thus largely exogenous to U.S. economic activity. Global mean temperatures, measured relative to a baseline level T^B—for instance, the pre-industrial level—satisfy: $\overline{T}_t - T^B = \epsilon\big(\sigma_t z_t + T_t\big)$.

The path T_t is deterministic and corresponds to a central climate scenario. The stationary stochastic process z_t captures uncertainty around the central warming scenario. We denote by $\mathcal{A}(z)$ its infinitesimal generator, the continuous-time equivalent of its transition matrix. We assume that z_t follows a continuous-time AR(1) process $dz_t = -\theta z_t dt + dW_t$. In this case, the operator $\mathcal{A}(z)$ is given by $\mathcal{A}(z)[V] = -\theta z \frac{\partial V}{\partial z} + \frac{1}{2}\frac{\partial^2 V}{\partial z^2}$. We assume that the aggregate shock dW_t is independent of the local shocks $dW_{\chi,it}$ and $dW_{\Delta,it}$. The deterministic path σ_t scales the shock and encapsulates the amount of uncertainty about future carbon emissions. For instance, when σ_t is constant, the ex-ante variance of global temperature around its deterministic path is constant over time. When σ_t scales with T_t, the standard deviation of global temperature increases proportionally with the level of temperature itself. ϵ is a scaling factor used to define the second-order approximation in our solution method.

The mean depreciation rate Δ_{it}, the productivity drift X_{it} and the two local volatilities depend on time through the path of global mean temperatures. We parameterize the means and the variances as linear functions of the temperature deviation, namely,

$$\begin{aligned}
\Delta_{it} &= \Delta_i + \epsilon\,\delta_i\big(T_t + \sigma_t z_t\big), \qquad X_{it} = \epsilon\,\kappa\,\chi_i\big(T_t + \sigma_t z_t\big), \\
\Sigma^2_{\Delta,it} &= \epsilon^2\,\Sigma^2_{\Delta,i}\big(\vartheta_{\Delta,i} + T_t + \sigma_t z_t\big), \qquad \Sigma^2_{\chi,it} = \epsilon^2\,\Sigma^2_{\chi,i}\big(\vartheta_{\chi,i} + T_t + \sigma_t z_t\big).
\end{aligned} \tag{5}$$

The slopes δ_i and χ_i are the damage functions for capital depreciation and productivity. The slope δ_i is the change in the annual depreciation rate per degree Celsius of warming. The slope χ_i is the change in the long-run level of log productivity per degree Celsius. Aggregate risk $\sigma_t z_t$ enters the

means and the variances alike.

The local volatilities $\Sigma_{\Delta,i}, \Sigma_{\chi,i}$ are local risk exposures: warming shifts the distribution of local weather, which increases the variance of economic fundamentals associated with costly storms and heat waves. The microfoundation in Appendix A delivers these functional forms when storms and heat waves of constant severity arrive at a Poisson probability that rises linearly with temperature, which is consistent with the data. This microfoundation implies that the variances are proportional to the absolute value of the slopes. For example, $\Sigma^2_{\Delta,i} = D^{\text{storm}}|\delta_i|$, where D^{storm} is the size of the capital depreciation shock of a single storm. $\vartheta_{\Delta,i}$ and $\vartheta_{\chi,i}$ capture the amount of local risk even absent climate change, due to the infrequent arrival of storms and heat waves at baseline.

2.4 Static Equilibrium

We now turn to the static equilibrium of our framework. The maximization problem of firms implies that the wage, w_{it}, in location i satisfies the firm's first-order condition $w_{it} = (1-\alpha)Z_{it}(S_{it}/N^P_{it})^{\alpha}$. Similarly, the rental rate for structures satisfies $r_{it} = \alpha Z_{it}(S_{it}/N^P_{it})^{-(1-\alpha)}$, since depreciation is not covered by the users of buildings. The optimal choice of labor in building production implies $\varpi r_{it}B_{it}/N^B_{it} = w_{it}$. Local demand for housing by workers implies that $\beta(1+s)\, w_{it}N_{it} = r_{it}H_{it}$.

We determine equilibrium prices and quantities as functions of the local capital stock K_{it} and local number of workers N_{it}. We show that labor is allocated to goods and buildings production in constant shares: $N^P_{it} = \varkappa^N N_{it}$ and $H_{it} = \varkappa^H B_{it}$, where the labor and building shares $\varkappa^N, \varkappa^H$ are combinations of parameters described in equation (B2), Appendix B.1. Using these shares and the production function of buildings, the wage and the rental rate of buildings become: $w_{it} = w_{i0}Z_{it}L_i^{\omega\alpha}(K^{1-\omega-\varpi}_{it}N^{\varpi-1}_{it})^{\alpha}$ and $r_{it} = r_{i0}Z_{it}L_i^{-\omega(1-\alpha)}(K^{1-\omega-\varpi}_{it}N^{\varpi-1}_{it})^{-(1-\alpha)}$, where the constants w_{i0}, r_{i0} are combinations of parameters expressed in Appendix B.1.

Profit maximization by developers implies that the rental rate of capital satisfies: $R_{it} = R_{0i}e^{\chi_{it}}K_{it}^{-\phi}N_{it}^{\psi} \equiv R_i(\chi_{it}, K_{it}, N_{it})$, where R_{0i} is a location-specific constant that depends on the permanent component of productivity and available land, and ϕ and ψ are combinations of parameters detailed in Appendix B.1.

We can now solve for consumption given prices. Consumption by workers in location i is equal to real income. It satisfies $\left(\frac{c_{it}}{1-\beta}\right)^{1-\beta}\left(\frac{h_{it}}{\beta}\right)^{\beta} = (1+s)\, w_{it}/r^{\beta}_{it} = c^w_{0i}e^{(1-\beta)\chi_{it}}\left(K^{1-\omega-\varpi}_{it}/N^{1-\varpi}_{it}\right)^{\xi}$, where c^w_{0i} is a location-specific constant, and ξ a combination of parameters detailed in Appendix B.1. r^{β}_{it} is the ideal price index in location i at time t, since we normalize the price of goods consumption to one and assume free trade. Finally, the flow utility of workers in location i at time t

is

$$U_i(a_{it}, \chi_{it}, K_{it}, N_{it}) \equiv u\left(A_{it}(1+s)\, w_{it}/r_{it}^{\beta}\right) = u\left(A_i c_{0i}^{w} e^{(\eta+(1-\beta))\chi_{it}} \left(K_{it}^{1-\omega-\varpi}/N_{it}^{1-\varpi}\right)^{\xi}\right).$$

2.5 Migration Decisions

Workers solve a forward-looking dynamic migration decision problem. We denote by V_{it} the value of being located in i at time t. V_{it} satisfies the Hamilton-Jacobi-Bellman equation

$$\rho V_{it} = U_i(a_{it}, \chi_{it}, K_{it}, N_{it}) + \mathcal{M}_i[V_t] + \mathbb{E}_t\left[\frac{dV_{it}}{dt}\right], \tag{6}$$

where we have already substituted standard discrete choice results to express the location maximization choice. In particular, the second term on the right-hand side is the continuation value from migration, namely: $\mathcal{M}_i[V_t] \equiv \mu\left\{\frac{1}{\nu}\log\left(\sum_j e^{\nu(V_{jt}-\tau_{ij})}\right) - V_{it}\right\}$. The last term in the worker's Bellman equation is the continuation value from changes in the aggregate states—fundamentals, population and capital in all locations—where the notation d denotes the time increment due to changes in aggregates only and not due to the individual migration choice.

Standard discrete choice results then imply that migration shares are a function of the vector of values, V_t, given by

$$m_{ij}(V_t) = \frac{e^{\nu(V_{jt}-\tau_{ij})}}{\sum_k e^{\nu(V_{kt}-\tau_{ik})}}. \tag{7}$$

The population distribution evolves according to the Kolmogorov Forward (KF) equation

$$\frac{dN_{it}}{dt} = \mu\left(\sum_k m_{ki}(V_t)N_{kt} - N_{it}\right). \tag{8}$$

We denote by $M^*(V) = \mu\big(m^*(V) - \mathbf{I}\big)$ the matrix that collects the transpose of the migration share matrix, m^*, so that the law of motion of the distribution (8) becomes, in matrix notation, $\frac{dN_t}{dt} = M^*(V_t)N_t$.

2.6 Investment Decisions

Capitalists solve a forward-looking dynamic investment problem. The two market structures differ in who bears the risk of local capital. We start with the diversified capitalist, whose problem delivers a standard asset-pricing characterization, and then turn to island capitalists.

2.6.1 Diversified Capitalists

Diversified capitalists hold capital K_{it} in every location and a bond B_t in zero net supply that pays the risk-free rate $\overline{R}_t$, determined by bond market clearing. They maximize $\mathbb{E}_0 \int_0^\infty e^{-\rho t} \overline{u}(C_t) dt$ subject to their budget constraint

$$C_t + \sum_i c_i(g_{it}) K_{it} + \dot{B}_t = \sum_i R_{it} K_{it} + \overline{R}_t B_t, \tag{9}$$

and the laws of motion (4) of the capital stock in all locations.

Denote by $\mathcal{Q}_t(B, \mathbf{K})$ the value of holding the bond B and the vector of capital stocks $\mathbf{K} = (K_1, \ldots, K_{\mathcal{I}})$ at time t. The Bellman equation of diversified capitalists is then

$$\begin{aligned} \rho \mathcal{Q}_t(B, \mathbf{K}) = \max_{C, \mathbf{g}} \overline{u}(C) + \dot{B}(C, \mathbf{g}, B, \mathbf{K}) \frac{\partial \mathcal{Q}_t}{\partial B} + \sum_i (g_i - \Delta_{it}) K_i \frac{\partial \mathcal{Q}_t}{\partial K_i} \\ + \frac{1}{2} \sum_{i,j} \Sigma_{\Delta,ijt} K_i K_j \frac{\partial^2 \mathcal{Q}_t}{\partial K_i \partial K_j} + \sum_{i,j} \Sigma_{\Delta,ijt} K_i \frac{\partial^2 \mathcal{Q}_t}{\partial K_i \partial \mathcal{K}_j} + \mathbb{E}_t \left[\frac{d \mathcal{Q}_t}{dt} \right], \end{aligned} \tag{10}$$

where $\dot{B}(C, \mathbf{g}, B, \mathbf{K})$ is defined from (9) and the last term holds individual states B and $\mathbf{K}$ fixed and collects the continuation value associated with the evolution of all aggregate states.

The first three terms are standard and would appear without risk. The first is the flow utility of consumption. The second is the continuation value from saving in bonds. The third term is the continuation value from net investment absent risk.

The fourth term encapsulates the value of local depreciation risk across the capitalist's portfolio. Because depreciation shocks are spatially correlated and capitalists face a joint investment problem, they value risk according to the entire spatial variance-covariance matrix $\Sigma_{\Delta,ijt}$ and to the concavity of their value function $\frac{\partial^2 \mathcal{Q}_t}{\partial K_i \partial K_j}$ with respect to their portfolio.

The fifth captures the covariance of a capitalist's private capital holdings with the overall capital stock of every location. Capital depreciation shocks affect equilibrium prices in all other locations j through changes in the local capital stock $\mathcal{K}_j$. These changes further affect the return to capital in all locations i. Investors value these joint changes through the concavity of their value function $\frac{\partial^2 \mathcal{Q}_t}{\partial K_i \partial \mathcal{K}_j}$, where the second derivative is now taken with respect to the equilibrium (log) capital stock $\mathcal{K}_j$ instead of their private capital stock K_j.

The last term captures changes in the aggregate states due to aggregate climate shocks, as well as local heat waves that do not affect the stock of capital directly.

Proposition 1. *(Diversified investment)*
In the diversified model, investment satisfies $g_{it} = c_{i0} Q_{it}^{\zeta}$*, where the price of capital* Q_{it} *solves*

$$0 = \overbrace{\frac{R_{it}}{Q_{it}} - \overline{R}_t + \frac{c_{i0}}{1+\zeta} Q_{it}^{\zeta} - \Delta_{it}}^{\text{excess net return}} - \underbrace{\overline{\gamma}\, \frac{1}{dt} \mathbb{C}ov_t \left[\frac{dK_{it}}{K_{it}} + \frac{dQ_{it}}{Q_{it}}, \frac{dC_t}{C_t} \right]}_{\text{own depreciation + GE capital gains risk}} + \underbrace{\sum_j \Sigma_{\Delta,ijt} \frac{1}{Q_{it}} \frac{\partial Q_{it}}{\partial \mathcal{K}_j}}_{\text{GE price hedge}} + \underbrace{\mathbb{E}_t \left[\frac{1}{Q_{it}} \frac{dQ_{it}}{dt} \right]}_{\text{aggregate states}},$$

and where aggregate capitalist consumption is $C_t = \sum_i \left[R_{it} - c_i(g_{it}) \right] K_{it}$*, and the risk-free rate satisfies* $\overline{R}_t = \rho + \frac{\overline{\gamma}}{dt} \mathbb{E}_t \left[\frac{dC_t}{C_t} \right] - \frac{\overline{\gamma}(\overline{\gamma}+1)}{2} \frac{1}{dt} \mathbb{V}ar_t \left[\frac{dC_t}{C_t} \right]$.

Proof. See Appendix B.2. □

Investment is determined by the price of capital, Tobin's Q. It satisfies a no-arbitrage equation: investing in the risk-free bond must deliver the same risk-adjusted expected return as investing in risky capital.

The first line captures the excess net return of capital: the dividend-price ratio (the rental rate) relative to the risk-free rate, the surplus from installing capital at price Q_{it}, net of expected depreciation. This surplus is positive because it encapsulates the benefit from investment net of installation costs. Given that investment costs satisfy an Inada condition, investing any positive amount generates inframarginal gains.

The second line captures the expected value of capital gains. The first component on the second line incorporates the risk premium associated with depreciation shocks and capital gains. As storms hit randomly, capital depreciates differently in different locations and time periods. The diversified investor values these shocks according to their covariance with their consumption process and their risk aversion: capital investments that pay off when consumption is low are more valuable. Similarly, fluctuations in the capital price are valued through their covariance with the capitalist's consumption.

The second component on the second line represents a general equilibrium adjustment. As capital depreciation shocks lower capital stocks across the U.S., equilibrium capital prices adjust at the same time as the investor's portfolio holdings depreciate. This co-movement leads to capital gains or losses. This component of capital gains does not depend on risk aversion because it depends on the covariance between capital holdings and equilibrium capital prices instead of on the covariance between capital prices and consumption. The third component encodes the

evolution of the aggregate states of the economy, as above. The risk-free rate is given by the usual Euler condition with a precautionary term.

The right-hand side of the equation in Proposition 1 defines the expected return on capital. Thus, and as we detail in Appendix D.1, the capital risk premium consists of the first component on the second line: $\overline{\gamma}\,\frac{1}{dt}\mathbb{C}\text{ov}_t\left[\frac{dK_{it}}{K_{it}}+\frac{dQ_{it}}{Q_{it}},\frac{dC_t}{C_t}\right]$.

2.6.2 Island Capitalists

We now describe the version of our framework without diversification, which we call the 'island model'. In the island model, capitalists in location i can only hold the local capital stock K_{it}, and no other asset. Therefore, they must self-finance all investment and cannot diversify. We denote by $\overline{C}_{it}$ their consumption, which satisfies the budget constraint $\overline{C}_{it}+c_i(g_{it})K_{it}=R_{it}K_{it}$, and by $\mathcal{Q}_{it}(K)$ the value of holding K units of local capital at time t.[2] Their Bellman equation is:

$$\begin{aligned}\rho\mathcal{Q}_{it}(K)=\max_g\,&\overline{u}\big([R_{it}-c_i(g)]K\big)+(g-\Delta_{it})K\frac{\partial\mathcal{Q}_{it}}{\partial K}\\&+\frac{\Sigma^2_{\Delta,it}}{2}K^2\frac{\partial^2\mathcal{Q}_{it}}{\partial K^2}+\sum_j\Sigma_{\Delta,ijt}K\frac{\partial^2\mathcal{Q}_{it}}{\partial K\partial\mathcal{K}_j}+\mathbb{E}_t\left[\frac{d\mathcal{Q}_{it}}{dt}\right].\end{aligned}\tag{11}$$

As for diversified capitalists, the first term is the flow utility of consumption after substituting out the budget constraint, and the second the continuation value from net investment. The third is the value of local depreciation risk, assessed through the concavity of the value function. The fourth is the covariance of the capitalist's own capital with the equilibrium capital stock of every location, which matters because prices depend on the capital distribution. We characterize investment in the island model using the following result.

Proposition 2. *(Island investment)*
In the island model, investment satisfies $c_i'(g_{it})\left[R_{it}-c_i(g_{it})\right]^{-\overline{\gamma}}=Q_{it}^{1-\overline{\gamma}}$, *where the certainty-equivalent value of capital,* Q_{it}, *satisfies*

$$\begin{aligned}0=&\frac{1}{1-\overline{\gamma}}\left[\frac{R_{it}-c_i(g_{it})}{c_i'(g_{it})}-\rho\right]+g_{it}-\Delta_{it}\\&-\frac{\overline{\gamma}}{2}\left\{\Sigma^2_{\Delta,it}+\frac{1}{dt}\mathbb{V}ar_t\left[\frac{dQ_{it}}{Q_{it}}\right]\right\}+(1-\overline{\gamma})\sum_j\Sigma_{\Delta,ijt}\frac{1}{Q_{it}}\frac{\partial Q_{it}}{\partial\mathcal{K}_j}+\mathbb{E}_t\left[\frac{1}{Q_{it}}\frac{dQ_{it}}{dt}\right].\end{aligned}$$

[2] We assume that capitalists, unlike workers, do not enjoy local amenities. This aligns their problem more closely with that of capitalists in the diversified model, where capitalists' location is indeterminate, and so local amenities play no role.

Proof. See Appendix B.3. □

The value Q_{it} is a certainty-equivalent price: holding K units of capital is worth consuming $\rho^{1/(1-\overline{\gamma})} Q_{it} K$ forever. Investment again equates the marginal adjustment cost to the price of installed capital in units of the final good, and has a unique solution $g_{it} = g_i(Q_{it}, R_{it})$.

The equation in Proposition 2 is again a no-arbitrage condition. Capital must deliver a return net of investment costs $R_{it} - c_i(g_{it})$ relative to the price of newly installed capital $c_i'(g_{it})$, compared with the rate of time preference. Net capital growth $g_{it} - \Delta_{it}$ raises the benefit of holding capital.

The three terms on the second line mirror those of Proposition 1. The first component represents capitalists' own depreciation risk and their GE capital gains risk. In the island model, we can solve explicitly for local capitalist consumption, so that the risk associated with their own capital depreciation is simply $\Sigma^2_{\Delta,it}$. Capital gains risk now only depends on the local variance of the value of capital, which also controls consumption.

The second component represents the now local GE price hedge. Local depreciation shocks affect the local value of capital in equilibrium, which partially insures owners against these shocks. This GE hedge depends on risk aversion in the island model because consumption depends directly on capital holdings, leading to the additional $-\overline{\gamma}$ term. The third component contains capital gains from changes in aggregate states.

2.7 Deterministic Steady State

In the deterministic steady state at baseline temperatures, $\epsilon = 0$, all time derivatives are equal to zero, and productivity is at its long-run level, $\chi_i^{ss} = 0$. The Bellman equation of workers (6) and the law of motion of population (8) become:

$$\rho V_i^{ss} = U_i(0, 0, K_i^{ss}, N_i^{ss}) + \mathcal{M}_i[V^{ss}], \tag{12}$$

$$0 = M^*(V^{ss}) N^{ss}, \tag{13}$$

where $N^{ss} = \{N_i^{ss}\}_i$ is the steady-state population distribution.

Both market structures share the same steady state. The investment rate, the rental rate of capital, and the risk-free rate of the diversified model satisfy:

$$g_i^{ss} = \Delta_i, \qquad R_i^{ss} = c_i(\Delta_i) + \rho\, c_i'(\Delta_i), \qquad \overline{R}^{ss} = \rho, \tag{14}$$

and the steady-state capital stock K_i^{ss} solves $R_i(0, K_i^{ss}, N_i^{ss}) = R_i^{ss}$, where the function R_i is defined

in Section 2.4 from the static equilibrium. Investment replaces depreciation, the rental rate covers the adjustment cost and the user cost of installed capital, and the risk-free rate of the diversified model equals the discount rate. Appendix B.4 describes a simple algorithm to solve for the steady state that uses equations (12)-(14).

Finally, Appendix H presents two extensions of our framework that we use in Section 5. The first consists in Epstein and Zin (1989) preferences for workers and capitalists, which separate risk aversion from the elasticity of intertemporal substitution. The second consists in within-location dispersion of storm damages across island capitalists. Equipped with our framework, we turn to the solution method.

3 Solution Method

3.1 Strategy

Our economy is a dynamic general equilibrium economy in which the distributions across locations of productivity $\chi_t = \{\chi_{it}\}_i$, workers $N_t = \{N_{it}\}_i$ and log capital $\mathcal{K}_t = \{\mathcal{K}_{it}\}_i$ are aggregate state variables that determine local prices. These distributions evolve according to the laws of motion (1), (8) and (4). The laws of motion of population and capital in turn depend on the values of workers, given by the Bellman equation (6), and on the value of capital. That value of capital is the price of capital in the diversified model, given by Proposition 1, and the value of capital in the island model, given by Proposition 2. We solve this dynamic fixed point problem by leveraging the 'Master Equation' representation of the economy (Bilal, 2023).

The Master Equation approach is structured in two steps. In the first step, we merge the individual decision problems with the laws of motion of the productivity, population, and capital distributions into a single Bellman equation: the Master Equation. The Master Equation is a state-space representation of the equilibrium. The second step consists in taking a local perturbation of the Master Equation in the scale parameter ϵ around a deterministic steady state of the economy.

3.2 The Master Equation

The values of workers and the value of capital depend on the aggregate state of the economy: the aggregate shock z_t, the distributions χ_t, N_t and $\mathcal{K}_t$, and calendar time through the warming path T_t. We therefore write $V_{it} = V_{it}(z_t, \chi_t, N_t, \mathcal{K}_t)$ and $Q_{it} = Q_{it}(z_t, \chi_t, N_t, \mathcal{K}_t)$, where the time subscript on V and Q now only captures the dependence on T_t. The core idea is that the three distributions are no more than large-dimensional state variables from the perspective of any given

agent. The chain rule then allows us to express the expected change of any function F of the state as its expected change along the laws of motion of every state variable. This operation defines the generator $\mathcal{L}_t$ of the economy—the operator that expresses expectations over small time intervals—given by

$$\begin{aligned}\mathcal{L}_t F \equiv & \underbrace{\frac{\partial F}{\partial t}}_{\text{change in } T_t} + \underbrace{\mathcal{A}(z)[F]}_{\text{change in } z_t} + \underbrace{\sum_j (X_{jt} - \kappa\chi_j)\frac{\partial F}{\partial \chi_j} + \frac{1}{2}\sum_{j,\ell}\Sigma_{\chi,j\ell t}\frac{\partial^2 F}{\partial \chi_j \partial \chi_\ell}}_{\text{change in } \chi_t\text{: drift and local risk}} \\ & + \underbrace{\sum_j \frac{\partial F}{\partial N_j}\left(M^*(V)N\right)_j}_{\text{change in } N_t} + \underbrace{\sum_j \left(g_{jt} - \Delta_{jt} - \frac{\Sigma^2_{\Delta,jt}}{2}\right)\frac{\partial F}{\partial \mathcal{K}_j} + \frac{1}{2}\sum_{j,\ell}\Sigma_{\Delta,j\ell t}\frac{\partial^2 F}{\partial \mathcal{K}_j \partial \mathcal{K}_\ell}}_{\text{change in } \mathcal{K}_t\text{: drift and local risk}}, \end{aligned} \tag{15}$$

where $g_{jt} = c_{j0}Q_{jt}^{\zeta}$ in the diversified model and $g_{jt} = g_j(Q_{jt}, R_{jt})$ in the island model. Population involves no local risk. Productivity and log capital do involve risk through the second derivatives of F weighted by $\Sigma_{\chi,t}$ and $\Sigma_{\Delta,t}$. The drift of log capital includes the Itô correction $-\Sigma^2_{\Delta,jt}/2$. Shifts in these distributions matter to residents of location i because they affect wages, building rents, and rental rates of capital in every location.

The Master Equation for workers replaces the expected change of V_{it} in the Bellman equation (6) by $\mathcal{L}_t V_i$. Similarly, the Master Equation for the price of capital does the same with Propositions 1 or 2. The next proposition states the equilibrium of the diversified model in terms of the Master Equations. We relegate the Master Equation for the price of capital in the island model to Appendix C.2.

Proposition 3. *(Equilibrium)*
An equilibrium of the diversified model is a pair of functions $V_{it}(z, \chi, N, \mathcal{K})$ and $Q_{it}(z, \chi, N, \mathcal{K})$ of the aggregate state that solves the Master Equations:

$$\begin{aligned}\rho V_{it} &= U_i\left(\eta\chi_i, \chi_i, e^{\mathcal{K}_i}, N_i\right) + \mathcal{M}_i[V] + \mathcal{L}_t V_i, \\ \rho Q_{it} &= R_i\left(\chi_i, e^{\mathcal{K}_i}, N_i\right) + \frac{c_{i0}}{1+\zeta}Q_i^{1+\zeta} - \Delta_{it}Q_i + \mathcal{L}_t Q_i + \mathcal{R}_{it} + \iota^Q_{it},\end{aligned}$$

together with the laws of motion of productivity (1), population (8), and capital (4). $\mathcal{R}_{it}$ denotes the response of the risk-free rate to changes in aggregate states and ι^Q_{it} collects the covariance terms of Proposition 1 and the second-order terms of the risk-free rate. Appendix C.2 expresses $\mathcal{R}_{it}$ and ι^Q_{it} in state-space form. In the island model, Q_i solves the Master Equation (C14) of Appendix C.2.

3.3 Expansion of the Master Equation

Although the Master Equations in Proposition 3 provide a parsimonious characterization of equilibrium, they are difficult to solve numerically because they depend on high-dimensional distributions. As a result, standard global methods quickly run into the curse of dimensionality. We instead follow Bilal (2023) and construct an analytic second-order perturbation. In particular, we seek a solution to the Master Equations in Proposition 3 when the scale parameter ϵ is sufficiently small. Namely, we define n, k and $\widehat{\chi}$ as the vectors of scaled deviations from the steady state of the population, log capital and productivity distributions, such that

$$N = N^{ss} + \epsilon n, \qquad \mathcal{K} = \mathcal{K}^{ss} + \epsilon k, \qquad \text{and} \qquad \chi = \epsilon \widehat{\chi},$$

where productivity is zero at the steady state since it is measured relative to a baseline.

To state our strategy as compactly as possible, we define $\times_m$ as the tensor product with respect to index $m \in \{1, 2, 3\}$, which generalizes matrix products. It takes as arguments a tensor (a three-dimensional array) and a matrix or vector. It returns the summation along the mth index of the tensor and the matching index of the matrix: the first index of the matrix for $\times_2$ and $\times_3$, and its second index for $\times_1$. For instance, for a tensor $\mathcal{V}_{ij\ell}$ and a vector n_j, the (i, ℓ) entry of $\mathcal{V} \times_2 n$ is $\sum_j \mathcal{V}_{ij\ell} n_j$. Similarly, the ith entry of $\mathcal{V} \times_3 k \times_2 n$ is $\sum_{j,\ell} \mathcal{V}_{ij\ell} n_j k_\ell$. We provide more details in Appendix C.1.

In addition, we index the three distributions by $x \in \{n, k, \chi\}$. For a generic index x, V^x denotes V^n, V^k or V^χ. Inside a formula, x also denotes the deviation vector n, k or $\widehat{\chi}$. For the worker problem, we then seek a solution of the form, in vector notation,

$$\begin{aligned} V_t(z, \epsilon\widehat{\chi}, N^{ss} + \epsilon n, \mathcal{K}^{ss} + \epsilon k) - V^{ss} = \epsilon \left\{ \sum_{x \in \{n,k,\chi\}} V^x x + v_t^z z + v_t^T \right\} \\ + \frac{\epsilon^2}{2} \Bigg\{ \sum_{x,y \in \{n,k,\chi\}} \mathcal{V}^{x,y} \times_3 y \times_2 x + 2 \sum_{x \in \{n,k,\chi\}} \left(V_t^{x,T} + V_t^{x,z} z\right) x \\ + v_t^{z,z} z^2 + v_t^{T,T} + 2 v_t^{z,T} z + v_t^{\Sigma,z} z + v_t^{\Sigma,T} + \overline{v}_t \Bigg\}. \end{aligned} \tag{16}$$

Similarly, we seek a solution for the price of capital, of the form,

$$Q_t(z, \epsilon\widehat{\chi}, N^{ss} + \epsilon n, \mathcal{K}^{ss} + \epsilon k) - Q^{ss} = \epsilon \left\{ \sum_{x \in \{n,k,\chi\}} Q^x x + q_t^z z + q_t^T \right\} \tag{17}$$
$$+ \frac{\epsilon^2}{2} \Bigg\{ \sum_{x,y \in \{n,k,\chi\}} \mathcal{Q}^{x,y} \times_3 y \times_2 x + 2 \sum_{x \in \{n,k,\chi\}} \left(Q_t^{x,T} + Q_t^{x,z} z\right) x$$
$$+ q_t^{z,z} z^2 + q_t^{T,T} + 2 q_t^{z,T} z + q_t^{\Sigma,z} z + q_t^{\Sigma,T} + \overline{q}_t \Bigg\}.$$

The perturbation reduces the problem to finding a finite set of coefficients. The first-order coefficients in the worker problem, $\{V^n, V^k, V^\chi, v_t^z, v_t^T\}$, and their capitalist counterparts, $\{Q^n, Q^k, Q^\chi, q_t^z, q_t^T\}$, are the responses of values and capital prices to the three distributions, to the aggregate shock, and to the trend. They constitute the First-order Approximation to the Master Equation (FAME). The second-order coefficients capture nonlinearities and cross effects of these responses, and the constant $\overline{v}_t$ is the ex-ante valuation of risk. They constitute the Second-order Approximation to the Master Equation (SAME). Substituting (16) and (17) into the Master Equations of Proposition 3 and identifying coefficients order by order delivers the FAME at first order and the SAME at second order.

3.4 The FAME

We first characterize the FAME, as it is a necessary step before evaluating the effect of risk in the SAME. We collect the first-order matrices in $\mathbf{V} \equiv \begin{pmatrix} V^n & V^k \\ Q^n & Q^k \end{pmatrix}$, $\mathbf{V}^\chi \equiv \begin{pmatrix} V^\chi \\ Q^\chi \end{pmatrix}$, and the vectors in $\mathbf{v}_t^z \equiv \begin{pmatrix} v_t^z \\ q_t^z \end{pmatrix}$, $\mathbf{v}_t^T \equiv \begin{pmatrix} v_t^T \\ q_t^T \end{pmatrix}$. We write $\mathbf{s}_t = (n_t, k_t, \widehat{\chi}_t)$ for the stacked population, capital and productivity deviations, and $\mathbf{I}$ for the identity matrix.

Proposition 4. *(FAME)*
The first-order derivatives of the value functions and the evolution of the distribution solve the quadratic matrix equation, the linear system and the linear ordinary differential equations:

$$\rho \mathbf{V} = \mathbf{D} + \mathbf{M}\mathbf{V} + \widetilde{\mathbf{I}}\left[\mathbf{V}\widetilde{\mathbf{M}} + \mathbf{V}\mathbf{G}\mathbf{V}\right], \qquad (\rho \mathbf{I} + \kappa\widetilde{\mathbf{I}})\mathbf{V}^\chi = \mathbf{D}^\chi + \overline{\Psi}\mathbf{V}^\chi,$$
$$(\rho \mathbf{I} + \theta\widetilde{\mathbf{I}})\mathbf{v}_t^z - \widetilde{\mathbf{I}}\,\partial_t \mathbf{v}_t^z = \Psi\sigma_t + \overline{\Psi}\mathbf{v}_t^z, \qquad \rho \mathbf{v}_t^T - \widetilde{\mathbf{I}}\,\partial_t \mathbf{v}_t^T = \Psi T_t + \overline{\Psi}\mathbf{v}_t^T,$$
$$\dot{\mathbf{s}}_t = \Theta \mathbf{s}_t + \Theta_t^z z_t + \Theta_t^T.$$

The matrices $\mathbf{D}$, $\mathbf{M}$, $\widetilde{\mathbf{M}}$, $\mathbf{G}$, $\mathbf{D}^{\chi}$ *and* $\widetilde{\mathbf{I}}$ *depend only on parameters and steady-state objects with closed-form expressions and are given in Appendix C.3.2. The continuation matrix* $\overline{\Psi}$*, the forcing* Ψ *and the transition* Θ *depend in addition on* $\mathbf{V}$ *and* $\mathbf{V}^{\chi}$*, and* Θ_t^z, Θ_t^T *on* $\mathbf{v}_t^z$, $\mathbf{v}_t^T$*. In the island model,* $\widetilde{\mathbf{I}} = \mathbf{I}$*, and* $\mathbf{D}^{\chi}$ *depends on* $\mathbf{V}$*.*

Proof. See Appendix C.3.2. □

The first equation of Proposition 4 is the deterministic FAME. It captures how the value function in each location responds to changes in the population and capital distribution. For instance, V_{ij}^n is the change in the value function in location i induced by an increase in population in location j.

The matrix $\mathbf{D}$ captures the direct impact of changes in the population and capital distribution on flow payoffs such as the utility function through changes in equilibrium prices. The term $\mathbf{MV}$ encapsulates changes in the option value of migration when the underlying population and capital distributions shift. In the diversified model, it also captures the first-order response of the risk-free rate which links prices in all locations. The terms $\mathbf{V}\widetilde{\mathbf{M}}$ and $\mathbf{VGV}$ represent two general equilibrium forces. After a deviation in the underlying distribution, the distribution will evolve going forward, leading to future deviations in prices and thus in values that in turn depend on optimal responses of all agents in the economy to this very deviation. The matrix $\widetilde{\mathbf{I}} = \mathbf{diag}(\mathbf{I}, \mathbf{I} - \mathrm{D}_{r\mu})$ differs from the identity only in the diversified model, where the risk-free rate responds to the expected change in capital prices through $\mathrm{D}_{r\mu}$ as we show in Appendix C.3.

The next three equations describe how value functions respond to changes in productivity in each location, to the aggregate shock, and to the deterministic trend. These Bellman equations are linear. In addition, the system of Bellman equations is block recursive: we solve the deterministic FAME first, then the productivity, trend and stochastic blocks.

Equipped with the solution to the FAME, we construct optimal migration and investment decisions. These optimal decisions allow us to construct the law of motion of the aggregate state variable $\mathbf{s}_t$ to first order, which collects the population, capital, and productivity distributions. The transition matrix Θ of these distributions has one block for each pair of distributions. Its top left block represents how the population distribution reacts to changes in population elsewhere, and is given by $M^* + GV^n$: population evolves according to the steady-state migration matrix M^* and according to the first-order response of migration shares to changes in value functions due to underlying changes in population, GV^n, where G encodes the response of migration to the values. Similarly, the vectors Θ_t^z and Θ_t^T have one block per distribution. For instance, the

population block of Θ_t^z represents how migration responds to the aggregate shock, and is given by Gv_t^z, the migration response to the loadings of the values on the shock. The trend vector Θ_t^T shares a similar structure. From the time path of the distribution, we construct all equilibrium variables in the first-order perturbation.

Of course, as a first-order perturbation, the FAME features certainty equivalence and cannot capture the effect of local and aggregate risk. We therefore proceed to a second-order approximation.

3.5 The SAME

The SAME consists of restrictions on the Hessian of the value functions, and follows a similar logic to the FAME. As a second-order perturbation, it incorporates the effect of local and aggregate risk. We collect the nine worker Hessians and the nine capitalist Hessians in the $2\mathcal{I} \times 3\mathcal{I} \times 3\mathcal{I}$ tensor $\mathcal{T} \equiv (\mathcal{V}^{x,y}, \mathcal{Q}^{x,y})_{x,y\in\{n,k,\chi\}}$, the cross matrices in the $2\mathcal{I} \times 3\mathcal{I}$ matrices $\mathbf{V}_t^z \equiv (V_t^{x,z}, Q_t^{x,z})_x$, $\mathbf{V}_t^T$, and the vectors in $\mathbf{v}_t^{z,z}, \mathbf{v}_t^{T,T}, \mathbf{v}_t^{z,T}, \mathbf{v}_t^{\Sigma,z}, \mathbf{v}_t^{\Sigma,T}, \overline{\mathbf{v}}_t$, where we always stack the worker block over the capitalist block.

Proposition 5. *(SAME)*
The second-order derivatives of the value functions and the second-order law of motion of the distribution solve the linear tensor Sylvester equation, the linear matrix Sylvester differential equations and the linear ordinary differential equations:

$$
\begin{aligned}
\rho\,\mathcal{T} &= \mathbb{T}_0 + \mathcal{T} \times_1 \Phi + \left[\mathcal{T} \times_2 \Theta + \mathcal{T} \times_3 \Theta\right] \times_1 \widetilde{\mathbf{I}},\\
(\rho\,\mathbf{I} + \theta\,\widetilde{\mathbf{I}})\,\mathbf{V}_t^z - \widetilde{\mathbf{I}}\,\partial_t \mathbf{V}_t^z &= \mathbf{V}_{0t}^z + \Phi \mathbf{V}_t^z + \widetilde{\mathbf{I}}\mathbf{V}_t^z\Theta,\\
\rho\,\mathbf{V}_t^T - \widetilde{\mathbf{I}}\,\partial_t \mathbf{V}_t^T &= \mathbf{V}_{0t}^T + \Phi \mathbf{V}_t^T + \widetilde{\mathbf{I}}\mathbf{V}_t^T\Theta,\\
(\rho\,\mathbf{I} + \theta_{\mathbf{v}}\,\widetilde{\mathbf{I}})\,\mathbf{v}_t - \widetilde{\mathbf{I}}\,\partial_t \mathbf{v}_t &= \mathbf{v}_{0t} + \Phi \mathbf{v}_t, \qquad \text{for any } \mathbf{v}_t \in \{\mathbf{v}_t^{z,z}, \mathbf{v}_t^{T,T}, \mathbf{v}_t^{z,T}, \mathbf{v}_t^{\Sigma,z}, \mathbf{v}_t^{\Sigma,T}, \overline{\mathbf{v}}_t\},\\
\dot{\mathbf{s}}_t &= \Theta \mathbf{s}_t + \Theta_t^z z_t + \Theta_t^T + \epsilon\left(\mu_{2t}^n, \mu_{2t}^k, 0\right),
\end{aligned}
$$

where $\theta_{\mathbf{v}}$ is defined as $\{2\theta, 0, \theta, \theta, 0, 0\}$ for the six vectors $\mathbf{v}$ respectively. The tensor $\mathbb{T}_0$ and the matrix Φ depend on the FAME and are given in closed form in Appendix C.4.2. The matrices $\mathbf{V}_{0t}^z$, $\mathbf{V}_{0t}^T$ and the vectors $\mathbf{v}_{0t}$ depend on the FAME and block-recursively on the blocks solved before them in the order listed. This dependence is given in closed form in Appendix C.4.2. The second-order drifts of the distribution, μ_{2t}^n and μ_{2t}^k, depend on the solution to the FAME and the SAME and are given in closed form in Appendix C.4.

Proof. See Appendix C.4.2. □

The structure of the Bellman equations in Proposition 5 mirrors the structure of those in Proposition 4. Both propositions characterize how value functions react to changes in underlying population, capital and productivity distributions, the aggregate shock, and time trends. The main difference between the SAME and the FAME is the dimensionality of the underlying value functions.

The first equation of Proposition 5 is the deterministic SAME. It is a linear symmetric Bellman equation which takes the form of a linear Sylvester tensor equation. We exploit this structure to propose a highly efficient algorithm to solve it with standard linear algebra techniques despite its high dimensionality.

The deterministic SAME encodes how the value function of workers and capitalists in any location changes when there is a joint deviation in the population, capital, and productivity distributions elsewhere, which affects values to second order according to $\mathcal{T}$. The tensor $\mathbb{T}_0$ collects the second derivatives of flow payoffs in population, productivity, and capital, and the products of first-order responses that enter because migration and investment are nonlinear in values and prices. The remaining terms capture how workers and capitalists anticipate the evolution of the underlying population, capital, and productivity distributions given that they are away from the steady state.

The following Bellman equations for $\mathbf{V}_t^z$ and $\mathbf{V}_t^T$ characterize how value functions respond to a joint deviation in the distribution and, respectively, a simultaneous aggregate shock or the trend. The Bellman equations for the nonlinear effect of an aggregate shock, the trend, or their interaction, $\mathbf{v}_t^{z,z}, \mathbf{v}_t^{T,T}, \mathbf{v}_t^{z,T}$, then depend block-recursively on all the previous value functions.

The vectors $\mathbf{v}_t^{\Sigma,z}$ and $\mathbf{v}_t^{\Sigma,T}$ contain the effect of rising local risk, through the covariance matrices $\Sigma_{\Delta,t}$ and $\Sigma_{\chi,t}$ interacted with the rest of the solution to the SAME. The vector $\overline{\mathbf{v}}_t$ contains the ex-ante valuation of aggregate risk through the coefficient of z^2. These three blocks depend on all the other blocks and therefore on the equilibrium response of population and capital to risk.

Equipped with the solution to the SAME, we construct the law of motion of the aggregate state to second order. The only difference with the first-order law of motion in Proposition 4 is the second-order drift. It contains two forces that are absent at first order. First, migration and investment respond to the second-order deviations in values and prices, including the valuation of local and aggregate risk. As a result, anticipated risk changes the paths of population and capital, unlike in the FAME. Second, migration flows and investment are nonlinear in the underlying population and capital distributions. The second-order drifts in the second-order law of motion encode this curvature. From the time path of the distribution, we construct all equilibrium variables in the

second-order perturbation.

Equipped with a solution method that captures nonlinearities and risk, we turn to the quantification of the model.

4 Model Estimation

4.1 Data

We assemble three types of data. First, we compile economic information at the county-year level from multiple sources. Employment, wages, and related statistics are drawn from the Bureau of Economic Analysis, while investment data at five-year intervals come from the Census of Manufactures. These datasets cover the period from 1960 to 2019.

Second, we construct a detailed weather dataset. We use daily data on surface temperature, wind speed, and precipitation on a 0.5° by 0.5° grid beginning in 1901 from the Inter-Sectoral Impact Model Intercomparison Project (ISIMIP). These reanalysis data combine historical station-level weather observations with climate model outputs to generate high-resolution weather series. We aggregate this information to build a dataset of annual weather extremes for each U.S. county and year. Appendix E describes the data sources in more detail.

Third, to quantify our model in a way that captures the economic impact of climate change, we draw on the reduced-form estimates in Bilal and Rossi-Hansberg (2025). Using a distributed lag specification that controls for a rich set of possible confounders in the same data as we use for quantification, they estimate the causal effects of storms and heat waves on income, employment, and investment at multiple horizons. We use these reduced-form impulse response functions as targets in our estimation. Figure E1 in Appendix E reproduces them.

4.2 Baseline Parameters

We start by determining baseline preference and production parameters. We estimate the migration elasticity of workers and the investment elasticity of capitalists in Section 4.4. Throughout the rest of the paper, we interpret an interval $[t, t+1)$ as one year.

We first determine preference parameters. We use a pure rate of time preference $\rho = 0.03$, which is standard in the climate change literature (Rennert et al., 2022). We set the housing expenditure share for workers to the common value of $\beta = 0.4$ following Diamond (2016). We set the risk aversion of workers to $\gamma = 2$ and that of capitalists to $\overline{\gamma} = 3$, within the range of 1 to 10 commonly used in the literature. Appendix G reports comparative statics in both parameters.

We set the rate of migration opportunities to $\mu = 2.30$ so that 90% of workers have the option to migrate within a year.

We calibrate the parameters in the production functions for goods and buildings to standard values. In the model, the share of commercial structures in goods production captures both capital and physical structures in the data. Standard estimates place these shares at 0.3 and 0.1, respectively, so we set $\alpha = 0.4$. To determine the labor share ϖ in building production, we use the share of employment in the structures production sector from BEA input-output tables, which equals 0.05. This share implies $\varpi = \frac{(1-\alpha)(1-0.95)}{0.95\alpha+(1-\alpha)\beta} = 0.05$. Finally, to calibrate the land share ω in building production, we rely on standard estimates from the real estate literature. For a representative urban property, land accounts for roughly 35% of total property value, with the remaining 65% corresponding to the building structure (Davis et al., 2021). We therefore set $\omega = 0.35$.

At baseline, we impose a common capital depreciation rate across counties, setting $\Delta_i = \Delta = 0.08$. Of course, climate change affects depreciation heterogeneously across locations. These location-specific effects are precisely the local damage functions we estimate below.

Finally, we set the parameters of the productivity and temperature processes. Productivity shocks mean-revert at rate $\kappa = 0.14$, which corresponds to a half-life of five years. Aggregate temperature shocks z_t mean-revert at rate $\theta = 0.1$, so that uncertainty about the warming trajectory persists for ten years on average. We calibrate the scale of aggregate risk $\sigma_t = \overline{\sigma} T_t$ so that the IPCC range of warming, 0 to 4°C around a central 2°C scenario, is the 95% band of our temperature distribution. This requires $\overline{\sigma}/\sqrt{2\theta} = 0.5$, that is, the standard deviation of $\sigma_t z_t$ equals $T_t/2$.

4.3 Inversion of Fundamentals

Our second step is to estimate time-invariant fundamentals for each location, taking as given estimates of the migration elasticity ν and the investment elasticity ζ.

We use 2012 data for wages w_i, employment N_i, investment I_i, and the migration flows $\mathtt{m}_{ij}$ of the 2012 to 2016 American Community Survey for every pair of counties i, j. We interpret 2012 as the initial steady state of the economy. We use a recursive scheme to recover the time-invariant fundamentals in every location, Z_i, A_i, c_{i0}, and τ_{ij}.[3] The following proposition shows that, given the data, these fundamentals can be uniquely recovered. We provide a formal proof in Appendix F.

[3]Because land rents are the same multiple s of the wage bill in every location and period, the inversion absorbs this adjustment into the underlying fundamentals. Thus, the rebate leaves our results unchanged conditional on inverting the model.

Proposition 6. *(Inversion of fundamentals)*
Given data w_i, N_i, I_i, π_{ij}, there exists a unique set of vectors of fundamentals, (Z_i, A_i, c_{i0}), and a unique symmetric migration cost matrix, τ_{ij}, that rationalize these data as the initial steady state.

The proof of Proposition 6 is constructive and therefore delivers an algorithm to recover the fundamentals. It extends standard inversion results from static environments to our fully forward-looking dynamic setting and follows closely the arguments in Bilal and Rossi-Hansberg (2025). The investment cost shifter c_{i0} follows from the steady-state conditions (14), which are the same in both market structures. Because we invert migration costs nonparametrically, however, we can recover only their symmetric component.[4]

4.4 Migration and Investment Elasticities

Our inversion result is derived conditional on the migration elasticity ν and the investment elasticity ζ. We estimate both internally, together with the amenity-productivity ratio η, using the reduced-form estimates of the impact of extreme events on economic activity of Bilal and Rossi-Hansberg (2025). We nest our inversion procedure in an outer loop in which we search over candidate values for ν, ζ and η.

To conduct our estimation exercise, we parameterize the effects of storms and heat waves in the model. Consistent with the reduced-form estimates, we assume that storms generate a capital depreciation shock of size D^{storm} in coastal counties. Heat waves generate a productivity shock D^{heat} and an amenity shock ηD^{heat} in warm counties which then recover at rate κ as in (1). Through the lens of our model, these shocks correspond to a single innovation to the depreciation and productivity Brownian motions of size D^{storm} and D^{heat}. Through the lens of the microfoundation in Appendix A, these shocks correspond to the realization of a Poisson event.

We construct the model counterparts of the event study estimates as follows. The impulse in county i is proportional to its number of storms or heat waves in the sample, consistent with the correlation structure of shocks in the data. The model event study is the difference between treated and control counties weighted by population as in the data. We estimate the elasticities and the damages with the diversified model and use them in both market structures.

The estimation proceeds in two stages. The first stage uses the FAME to deliver preliminary estimates and propose efficient global identification guarantees by exploiting a key property of the linear solution. By construction, the response of the economy is linear in the size of the shock,

[4]Without symmetry, migration costs are not separately identified from local amenities. If, instead, we imposed that migration costs depend on a distance metric, this dependence could be estimated flexibly.

so that relative impulse responses do not depend on the magnitude of a common shock. Once we target relative impulse responses, we can therefore normalize the shock magnitude arbitrarily and estimate ν, ζ and η without pinning down D^{storm} and D^{heat}. We simulate the model for a large number of parameter vectors drawn from a Sobol sequence. For each vector, we invert fundamentals as in Section 4.3, solve for the steady state, compute the FAME, and simulate the response of the economy to a storm and a heat wave. We then apply a root solver to refine the estimates of these three parameters.

The second stage uses the SAME and delivers our final estimates. Appendix C.5 gives the second-order expansions of wages and investment behind the model responses. Under the SAME, impulse responses vary quadratically with the magnitude of the shock, so that relative responses no longer isolate the elasticities from the shock sizes. We turn this quadratic property to our advantage. Proposition 5 implies that the cumulative impulse response (CIR) of any outcome to a shock of scale D is exactly quadratic in D. One SAME evaluation at a trial scale D, together with the FAME, identifies the whole function:

$$\text{CIR}(s) = d_1 s + d_2 s^2, \qquad d_1 = \frac{\text{CIR}^{\text{FAME}}(D)}{D}, \qquad d_2 = \frac{\text{CIR}^{\text{SAME}}(D) - \text{CIR}^{\text{FAME}}(D)}{D^2},$$

where CIR denotes the ten-year cumulative impulse response of the targeted outcome. The size of the shock is the root of this quadratic that matches the level of the response in the data. In practice, the nonlinear estimates are close to the linear ones.

Figure 1 summarizes the first stage, in which we solve the model for 200 candidate triplets (ν, ζ, η) and compare the model-implied moments with their empirical counterparts.

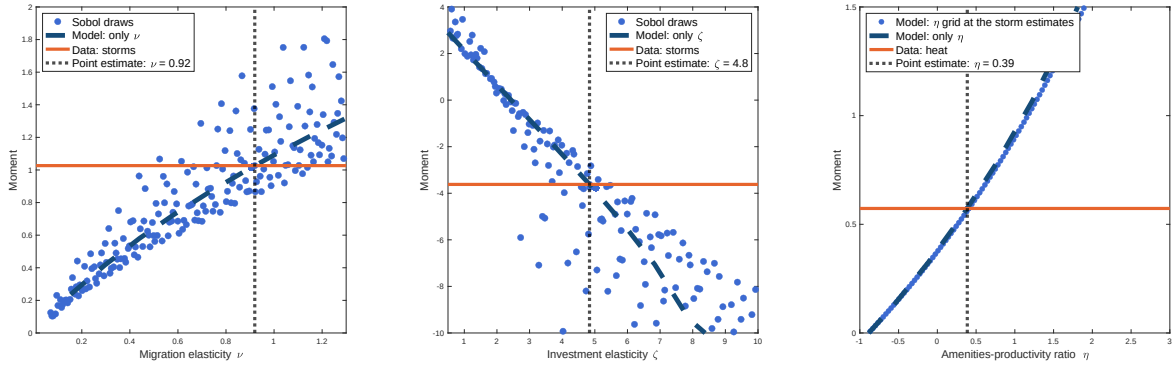
Although we estimate the three parameters (ν, ζ, η) jointly, we briefly discuss how we select each moment to provide intuition for identification. The gravity structure of migration implies that the migration elasticity ν is identified by the response of local employment relative to that of income per capita, holding amenities fixed. Accordingly, we use the response of employment relative to income per capita following a capital depreciation shock, which does not affect amenities. Figure 1(a) plots this moment. Each blue dot corresponds to a pair (ν, ζ) of the first stage, ordered on the x-axis by ν. The scatterplot reveals a clear positive relationship: across all values of ζ, higher values of ν are associated with a stronger response of employment relative to income per capita. The same pattern holds when we fix ζ at its point estimate, which illustrates univariate as well as multivariate identification. The orange horizontal line corresponds to the empirical moment for storms in coastal counties.

Figure 1: Identification of the migration elasticity, the investment elasticity, and the amenity-productivity ratio.

(a) Storms: employment/income p.c. CIR

(b) Storms: investment/employment CIR

(c) Heat waves: employment/income p.c. CIR



Note: Blue dots in (a) and (b): 200 Sobol draws of (ν, ζ) in $[0.01, 1.3] \times [0.5, 10]$, storm shock, linear (FAME) stage. Blue dots in (c): η on a grid in $[-1, 3]$ at the storm estimates of (ν, ζ). Dashed dark blue line: moment in the model when only the parameter on the x-axis varies, other parameters at their estimates. Solid orange line: moment in the data (storms in coastal counties in (a) and (b), heat waves in warm counties in (c)). Dotted vertical line: point estimate.

The investment elasticity ζ is informed by the cumulative response of investment relative to employment ten years after a capital depreciation shock, which we target with the response to storms in coastal counties. Intuitively, the employment response captures the overall size of the shock, while the response of investment relative to employment reflects how strongly investment reacts to the price of installed capital. Figure 1(b) plots this moment against ζ. The moment falls monotonically with ζ, both across the Sobol draws and when ν is fixed at its point estimate. When adjustment costs are steep, the investment rate barely responds to the price of installed capital, so investment falls with the stock and the ratio is positive. At the estimate, investment rises while employment falls, so the ratio is negative and grows in absolute value with ζ. If investment responds strongly relative to employment, adjustment costs must be mild, which corresponds to a higher value of ζ.

To inform the amenity-productivity ratio η, we use the cumulative response of employment relative to income per capita ten years after a productivity-amenity shock. Our estimation is block-recursive, and we can identify η in a univariate search in a second step, which is why the corresponding scatterplot is a single line. We match this moment to its empirical counterpart following a heat wave in warm counties. Figure 1(c) plots this moment against η, holding (ν, ζ) at their storm estimates. The moment rises steeply with η: the more amenities fall together with productivity, the more workers leave relative to the decline in income per capita. The empirical target lies above

the response of the model to a pure productivity shock. Given ν, employment therefore responds more strongly than a productivity shock alone implies, so that amenities must fall together with productivity: the estimate of η is positive.

Our indirect inference approach implies that the migration elasticity is $\nu = 0.92$. This value is higher than the value of 0.2 reported by Caliendo et al. (2019) in their state-level specification. Given that our spatial unit is a county, a higher elasticity is reasonable. Here, we also obtain an estimate somewhat larger than in Bilal and Rossi-Hansberg (2025), largely due to the difference in worker risk aversion and the diversified investment structure. We also find that amenities respond $\eta = 0.39$ times as much as productivity following a heat wave. Amenities move in the same direction as productivity, which amplifies the employment response. Finally, the investment elasticity is $\zeta = 4.8$: a one percent higher price of installed capital raises the investment rate by 4.8%. This estimate is similar in magnitude to the one in Bilal and Rossi-Hansberg (2025).

4.5 Damage Functions and Local Risk Exposures

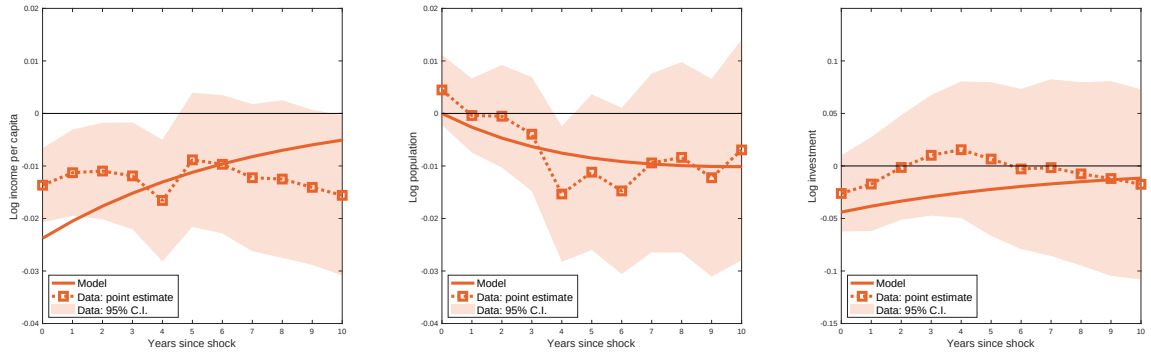
With the migration and investment elasticities in hand, we turn to estimating the damage functions for capital depreciation and productivity, δ_i and χ_i in (5), the local risk exposures $\Sigma_{\Delta,i}$ and $\Sigma_{\chi,i}$, and the spatial correlations of the shocks, $\mathfrak{c}_{\Delta,ij}$ and $\mathfrak{c}_{\chi,ij}$. We proceed in two steps. First, we obtain the common economic damages from a single storm, D^{storm}, and from a single heat wave, D^{heat}, as a byproduct from the procedure in Section 4.4. Second, we interact these damages from single events with local changes in the probability that these events occur as global mean temperature rises.

In the first step, we obtain the damages from a single storm and from a single heat wave by matching the level of the cumulative impulse responses. This step uses the quadratic property of the SAME of Section 4.4. For heat waves in warm counties, we target the response of income per capita. Since the estimated model matches the response of employment relative to income per capita exactly in Figure 1, the model also matches exactly the cumulative responses of both employment and income per capita at the 10-year horizon. For storms in coastal counties, we target the cumulative effect on employment ten years after the shock. Because the estimated model matches the response of investment relative to employment exactly in Figure 1, the model also matches exactly the cumulative response of investment at that horizon.

Figure 2 compares the impulse responses of income per capita, employment, and investment following heat waves in warm counties. The model reproduces the decline in income per capita and employment over the decade. Income per capita falls at once in the data and in the model.

Figure 2: Heat waves: impulse responses in model and in data.

(a) Income per capita (b) Employment (c) Investment



Note: Impulse responses for income per capita (a), employment (b), and investment (c) after a 1-in-20-year heat wave in a warm county, in the model solved at the county level (3,037 counties). For (a) we use the response of wages, w_{it}, in the model and income per capita in the data. Solid line: model. Dotted line with square markers: data point estimate. Shaded area: data 95% confidence band. The 10-year cumulative impulse responses for income per capita (a) and employment (b) are targeted. The impulse response for investment (c) is untargeted.

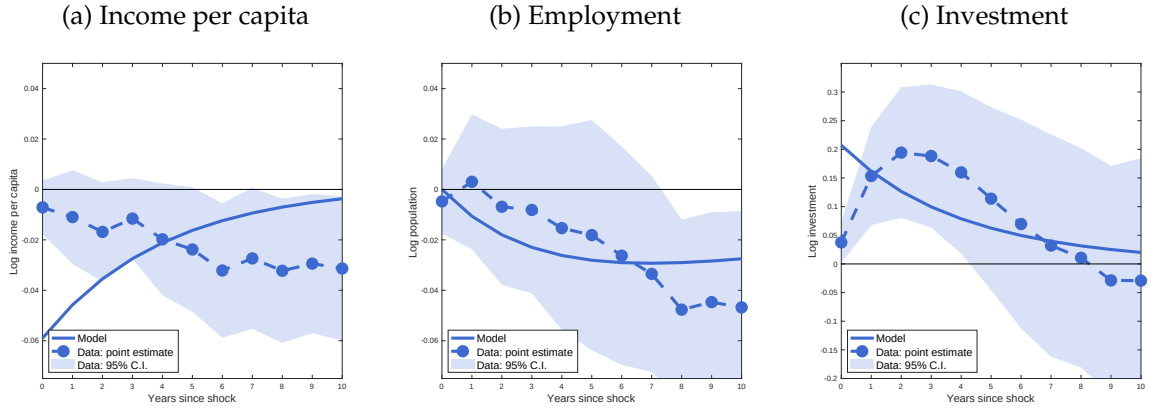
The drop on impact is somewhat larger in the model and recovers faster. The employment response builds gradually to about 1% after ten years and tracks the data closely. The response of investment, which is not targeted, is negative throughout in the model, at 1 to 4%, and lies within the confidence band of the data. The data point estimate starts negative, turns slightly positive in the middle of the decade, and ends near the model.

Our impulse response matching exercise implies that a 1-in-20-year heat wave lowers log productivity by $D^{\text{heat}} = 2.4\%$ on impact in a warm county and lowers amenities by $\eta \times D^{\text{heat}} = 0.9\%$. While economically meaningful, the productivity loss is smaller than the local impacts estimated for the world as a whole, including developing countries.[5]

Figure 3 compares the impulse responses following storms in coastal counties. Employment falls gradually in the model and in the data, although the decline is front-loaded in the model. It reaches about 3% after six years and then flattens, while the data keep falling to almost 5% after eight years. Investment rises on impact in the model, by about 20%, and decays over the decade, whereas the data rise more slowly and peak at a similar level in the second and third years. Both paths lie within the 95% confidence band at all horizons but the impact year of investment. The fit is weaker for income per capita. The model predicts a drop of about 6% on impact followed by a fast recovery. The data instead show a gradual and persistent decline to about 3%, so that the

[5]For instance, Cruz and Rossi-Hansberg (2024) find that, in the warmest locations, a 1°C increase in winter temperatures can lower productivity by more than 15%.

Figure 3: Storms: impulse responses in model and in data.



Note: Impulse responses for income per capita (a), employment (b), and investment (c) after a 1-in-50-year storm in a coastal county, in the model solved at the county level (3,037 counties). For (a) we use the response of wages, w_{it}, in the model and income per capita in the data. Solid line: model. Dashed line with circle markers: data point estimate. Shaded area: data 95% confidence band. The 10-year cumulative impulse responses for income per capita (a), employment (b), and investment (c) are targeted.

model lies outside the confidence band in the first two years after the storm.[6]

Our impulse response matching exercise implies that a 1-in-50-year storm generates a capital depreciation shock of $D^{\text{storm}} = 26\%$ in a coastal county, close to the 30% of Bilal and Rossi-Hansberg (2025).

We now turn to the second step of the estimation strategy, which consists of constructing damage functions that vary with global mean temperature. This part of the analysis closely follows Bilal and Rossi-Hansberg (2025), but we include it here for completeness. We combine our estimated shock magnitudes, $(D^{\text{storm}}, D^{\text{heat}}, \eta D^{\text{heat}})$, with long-run changes in the probability of the corresponding extreme events.

To estimate how the probability of extreme events evolves across locations, we use the historical weather record over the last century. For each county i and event type $e \in \{\text{storm}, \text{heat}\}$, we regress the yearly indicator $\mathbb{1}_{it}^{e}$ that the event occurs on a constant and global mean temperature $\overline{T}_t$ over the period 1920 to 2019. The specification reads:

$$\mathbb{1}_{it}^{e} = \text{cst}_i^{e} + p_i^{e}\,\overline{T}_t + u_{it}^{e}. \tag{18}$$

The slope p_i^e is the change in the annual probability of the event per degree of warming. Equation (18) delivers a statistical downscaling, a standard approach in the climate science and economics literature (Cruz and Rossi-Hansberg, 2024), which links local event probabilities to global mean temperature. Figure E2 in Appendix E plots the frequency of both events against global tempera-

[6] This could be the result of forces outside the model, like government interventions and other forms of aid.

ture. We estimate the slopes for 100 clusters of counties to limit measurement error.

We then construct mean damage functions and local risk exposures by matching the first and second moments of the depreciation and productivity process implied by the hazard rate in (18). Specifically, a storm has an (excess) probability $p_i^e(\overline{T}_t - T^B)$ of occurring each year and causes D^{storm} depreciation when it occurs. In continuous time, either no event or a single event occurs in any given time period. In expectation, the excess capital depreciation rate is thus simply $p_i^e(\overline{T}_t - T^B)\ D^{\text{storm}}$ because excess storms follow a binomial distribution. Similarly, the excess variance of capital depreciation is $p_i^e(\overline{T}_t - T^B)\big(1 - p_i^e(\overline{T}_t - T^B)\big)\ (D^{\text{storm}})^2$, which converges to $p_i^e(\overline{T}_t - T^B)\ (D^{\text{storm}})^2$ in continuous time. Our microfoundation with Poisson processes in Appendix A makes this logic precise.

Applying these arguments to storms and heat waves, we thus obtain the damage functions:

$$\begin{aligned} \delta_i &= \mathbb{1}\{i \text{ coastal}\}\ p_i^{\text{storm}} D^{\text{storm}}, \qquad \chi_i = -\mathbb{1}\{i \text{ warm}\}\ p_i^{\text{heat}} \frac{D^{\text{heat}}}{\kappa}, \\ \Sigma_{\Delta,i}^2 &= D^{\text{storm}} |\delta_i|, \qquad \Sigma_{\chi,i}^2 = \kappa\, |\chi_i|\, D^{\text{heat}}, \end{aligned} \tag{19}$$

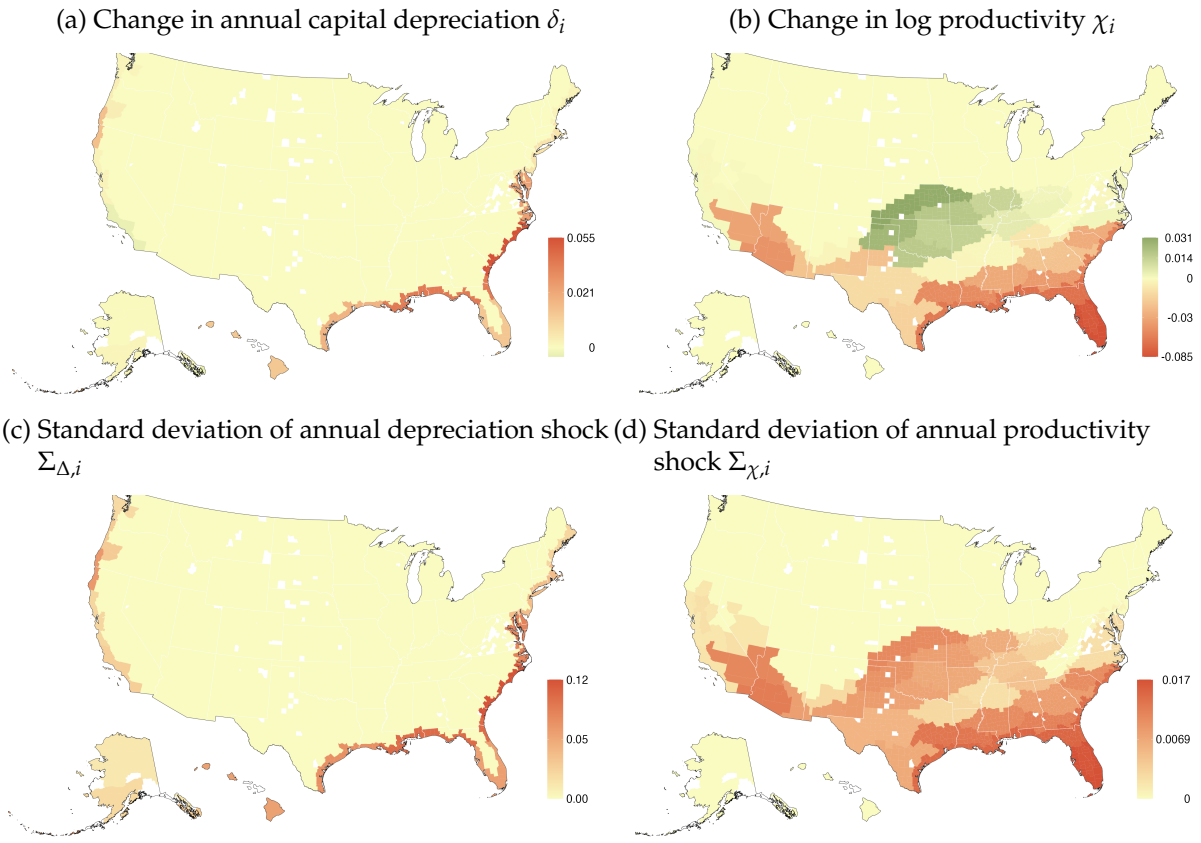
where division by κ cumulates the lagged effects of heat waves and converts the estimate into the long-run effect of a permanent increase in exposure to extreme heat. We estimate the spatial correlation of storms and heat waves $\mathfrak{c}_{\bullet,ij}$ from the correlation across clusters of the yearly indicators net of their temperature trend over 1920 to 2019.[7] Appendix E.4 details the estimator.

Figure 4 maps the estimated damage functions and risk exposures across U.S. counties. Figure 4(a) shows the effect of a 1°C increase in global mean temperature on the annual capital depreciation rate. Storms raise depreciation the most along the Gulf coast from Louisiana to the Florida panhandle and along the Atlantic coast of Georgia and the Carolinas. There, the annual depreciation rate rises by up to 5.5 p.p. per degree. This effect represents 0.2 additional storms per year per degree, each destroying 26% of the stock. As we do not detect economically meaningful effects of storms in inland counties in the data, storm damages affect a much smaller fraction of counties than heat waves do.

Figure 4(b) shows the long-run effect of a 1°C increase in global mean temperature on log productivity. These effects are highly heterogeneous across space. Losses are largest in Florida along the Gulf coast and on the southern Atlantic coast, where they reach 8.5% per degree in the most exposed counties. They reach about 4% in the deserts of Arizona and southern California. In contrast, the central Plains, from Kansas to Oklahoma and east into Missouri, experience gains of

[7] We convert these cluster-level correlations to the county level using the within-cluster correlation.

Figure 4: Damages and risk exposures for a 1°C increase in global temperature.



Note: Effect of a 1°C increase in global mean temperature: county-level maps of the cluster-level slopes (3,037 counties, 100 clusters). Panel (a): δ_i, change in the annual capital depreciation rate. Panel (b): χ_i, long-run change in log productivity. Panel (c): $\Sigma_{\Delta,i} = \sqrt{D^{\text{storm}}|\delta_i|}$, standard deviation of the annual depreciation shock. Panel (d): $\Sigma_{\chi,i} = \sqrt{\kappa|\chi_i|D^{\text{heat}}}$, standard deviation of the annual productivity shock. Color scales are in annual rates for capital and in log points for productivity, so that 0.055 in (a) is 5.5 p.p. and -0.09 in (b) is a 9% loss. Amenity changes are $\eta\chi_i$ and of the same sign.

up to 3% per degree, consistent with a historical decline in heat wave exposure in those locations. By assumption, amenity changes are proportional to productivity changes and of the same sign, so we omit a separate map for amenities. These damage functions for levels are similar to the ones in Bilal and Rossi-Hansberg (2025).

The bottom row maps local risk exposures, which are, we believe, novel to the literature. The standard deviation of the annual capital depreciation shock in Figure 4(c) shares the geography of expected depreciation. It reaches 11.9 p.p. at one degree of warming in the most exposed coastal county, against an average of 0.5 p.p. across counties. The standard deviation of the annual productivity shock in Figure 4(d) reaches 1.7% at one degree of warming in Florida and along the Gulf coast, against an average of 0.4%. Because the volatility scales with the absolute value of the slope, excess volatility arises in the South and the central Plains alike. Through the lens of our

structural model, we can now compare the importance of these local risks with that of aggregate risk, and the cost of both with that of the warming trend.

5 The Impact of Climate Risk in the United States

5.1 Setup

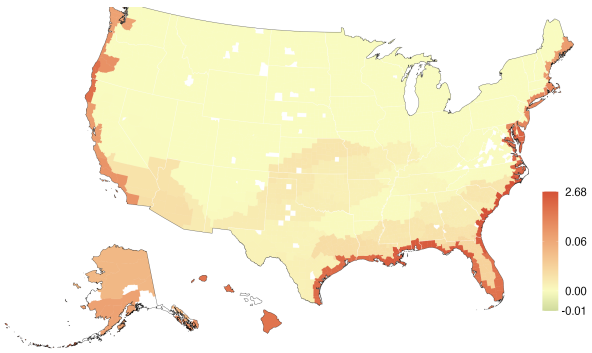
We evaluate the effects of aggregate and local risk on economic activity across the United States. We take 2025 as the year in which agents learn the warming path T_t and the paths of aggregate and local risk. Temperature rises along a concave path, by 1.9°C in 2100 and 2°C in 2125, and remains constant thereafter.

We report two exercises. The first isolates the geography of risk. The economy starts at the deterministic steady state. We then permanently switch on the local risk associated with climate change, the associated aggregate risk, or both. We set the magnitude of that risk to what it would be in a 2°C warmer climate, but do not include the corresponding trend. We then compare the new stochastic steady state with the deterministic one, county by county. The second exercise follows the transition from 2025 to 2100 under two configurations: the trend alone and the trend with local and aggregate risk.[8]

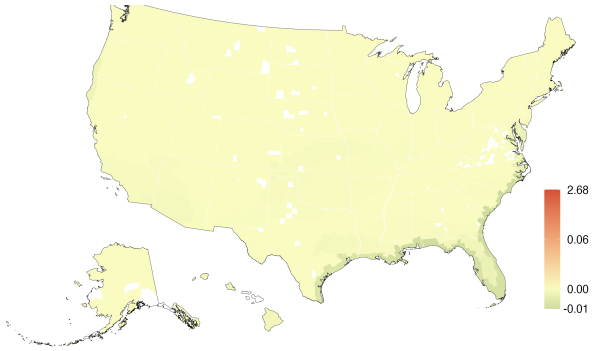
We report capital risk premia and consumption-equivalent welfare. The capital risk premium is the expected excess return on installed capital over the risk-free rate in the diversified model, and over the shadow rate of the local owner in the island model.[9] To measure the insurance provided by equilibrium price responses, we also solve each model without the covariance between the price of capital and local shocks in the valuation of capitalists. We call this calculation the no-covariance valuation, labeled 'no covariance' in the figures and detailed in Appendix D.3. We occasionally summarize results by grouping counties into coastal, inland warm, and inland cold locations.

Figure 5: Capital risk premium from local and aggregate risk (island model).

(a) Local risk: capital risk premium (% per year)



(b) Aggregate risk: capital risk premium (% per year)



Note: Island model. Shadow risk premium on installed capital, in percent per year, in the stochastic steady state with the local risk (a) or the aggregate risk (b) of a 2°C warmer climate, relative to the deterministic steady state of that climate. The two panels share the color scale. The aggregate-risk premium is zero or slightly negative in every county, at most 0.006% per year in absolute value.

5.2 Local Risk Dominates Aggregate Risk in Any Year

We start by examining risk premia in the island model, which serves as a yardstick for the magnitude of raw risk. In this model there is no diversification since capitalists cannot invest in other assets. Figure 5(a) maps the premium generated by local risk in the island model. It reaches up to 2.7 p.p. per year on the Gulf and southeast Atlantic coasts. It is also high along the rest of the Atlantic coast up to New England, along the Pacific coast, most of all north of San Francisco, in Hawaii, and in Alaska. A faint band over the Plains reflects the productivity risk of heat waves. Elsewhere inland the premium is essentially zero.

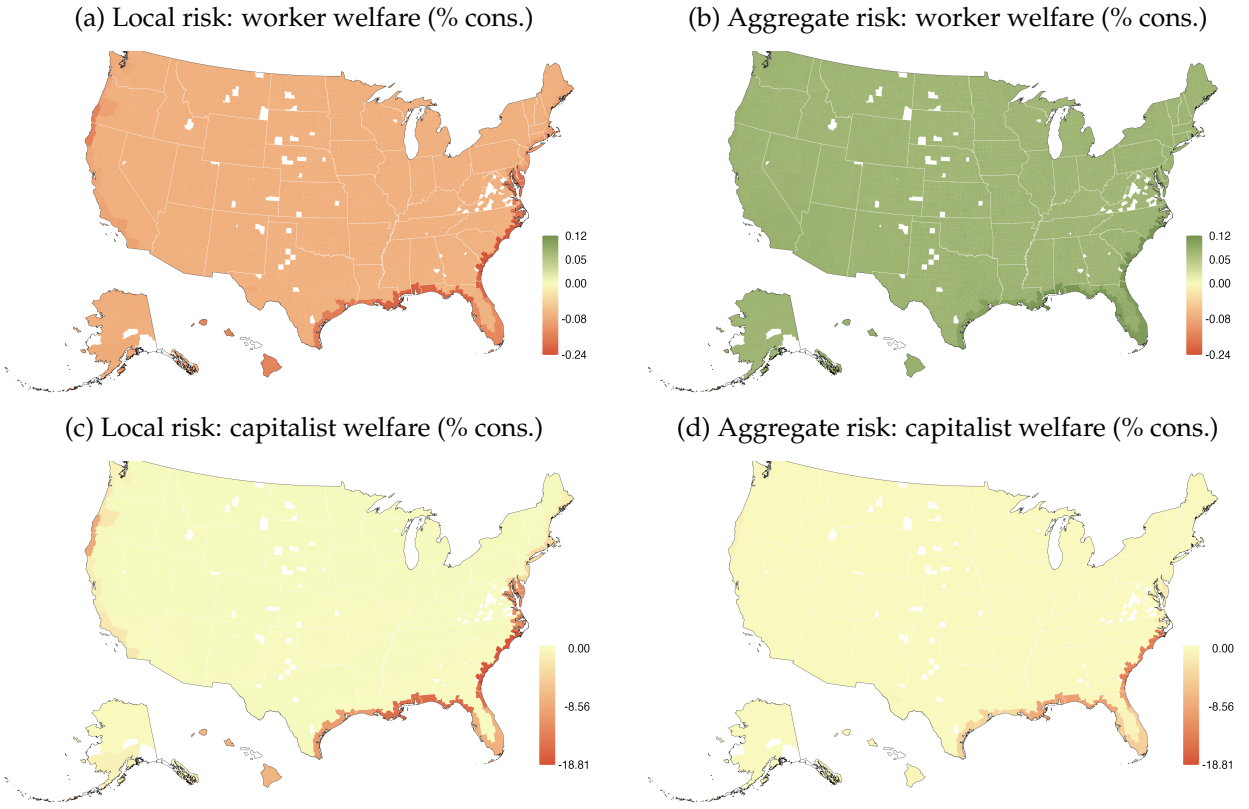
By contrast, aggregate risk generates negligible capital risk premia. They are even slightly negative because of the Oi-Hartman-Abel effect: aggregate risk is persistent, creating a positive investment option value. Persistence implies that investors can intertemporally arbitrage between good and bad years, investing more in high-return periods and less in low-return periods. This arbitrage increases expected returns and thus consumption relative to an economy without any risk at all.

Why does local risk dominate? Consider the variance of the depreciation rate that an investor

[8] Since baseline risk and the risk induced by warming are additive up to second order, these comparisons are isomorphic to comparing the stochastic economy with baseline risk and warming risk to the stochastic steady state with only baseline risk. In practice and for precision, we solve the transition with several expansion points: besides the initial and final steady states, we use six intermediate steady states (that are never actually reached) corresponding to fundamentals evenly spaced between 0 and 2°C of warming. Along the transition, we use the expansion around the steady state whose climate is closest to each calendar year and paste consecutive solutions.

[9] See the general forms in Appendix D.1.

Figure 6: Welfare effects of local and aggregate risk (island model).



Note: Island model. Lifetime consumption-equivalent welfare change, in percent, of workers (top row) and island capitalists (bottom row) in the stochastic steady state with the local risk (left) or the aggregate risk (right) of a 2°C warmer climate, relative to the deterministic steady state of that climate. Each row shares its color scale.

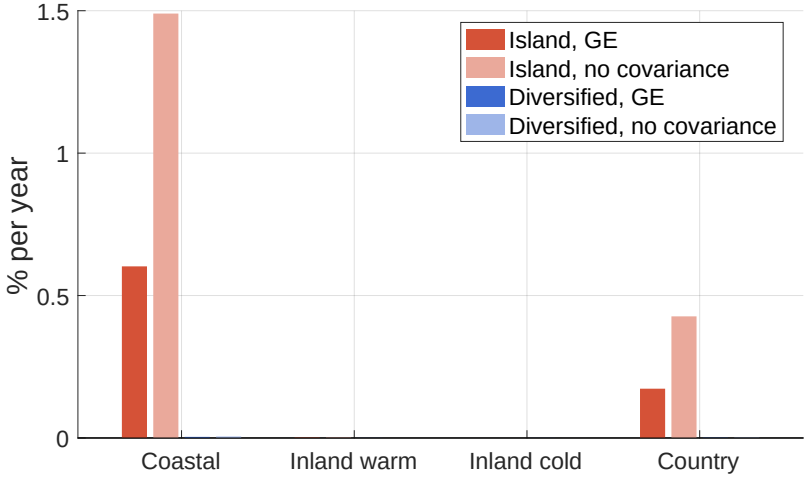
faces. We compute it from (2). It reads:

$$\mathbb{V}\mathrm{ar}_0\left[d\overline{\Delta}_{it}\right] \;=\; \underbrace{\Sigma^2_{\Delta,i}T_t}_{\text{local}} + \underbrace{\frac{\delta_i^2\sigma_t^2}{2\theta}}_{\text{aggregate}} \;=\; (D^{\text{storm}})^2\Big(p_i^{\text{storm}}T_t + \frac{\overline{\sigma}^2}{2\theta}\,(p_i^{\text{storm}})^2T_t^2\Big). \tag{20}$$

Rising storm probabilities raise the mean and the variance of depreciation in proportion to $p_i^{\text{storm}}T_t$: local risk shifts the whole distribution of storms every year. Aggregate risk operates only through the average depreciation rate $\mathbb{E}[\Delta_{it}|z_t]$, which already depends on small storm probabilities. Thus, the contribution of aggregate risk includes the small extra factor $(\overline{\sigma}^2/2\theta)\,p_i^{\text{storm}}T_t$. In the most exposed county, the annual storm probability rises by 0.2 per degree, so $p_i^{\text{storm}}T_t = 0.4$ at 2°C and the aggregate share of the variance is below one tenth, and is smaller everywhere else.

Aggregate risk is quantitatively relevant for welfare, however. The bottom row of Figure 6 maps the welfare of capitalists in the island model. Local risk costs up to 19% of lifetime con-

Figure 7: Capital risk premium with and without the price covariance, by location group.



Note: Capital risk premium in 2100 along the 2°C transition with all risk, in percent per year, capital-weighted within each group. Island model in red, diversified model in blue. Dark bars: general equilibrium. Light bars ('no covariance'): premium without the covariance between the price of capital and local shocks (Appendix D.1). Coastal: counties with a coastal share of at least one half. Inland warm and cold: split at the population-weighted median temperature.

sumption on the Gulf and southeast Atlantic coasts (Figure 6(c)): 3.0% in the coastal group and 0.7% for the country. Aggregate risk costs up to 14% on the same coasts (Figure 6(d)): 0.6% in the coastal group and 0.3% for the country. The reason is that persistence matters more for welfare than for the risk premium. The variance of aggregate risk in any year, and hence the risk premium, is small. The lifetime effect of decades-long persistent uncertainty on an investor who cannot diversify coastal capital is large.

Workers are affected through changes in the capital stock. Two effects push in opposite directions. On the one hand, risky depreciation gradually erodes the capital stock. On the other hand, risk leads to precautionary savings. For local risk, the first effect dominates, while for aggregate risk, the second effect dominates. The top row of Figure 6 illustrates these patterns: workers lose between 0.1 and 0.2% of lifetime consumption from local risk in every county, but gain 0.1% from aggregate risk.[10] Having mapped the geography of climate risk in the U.S., we now assess how the economy adjusts to it.

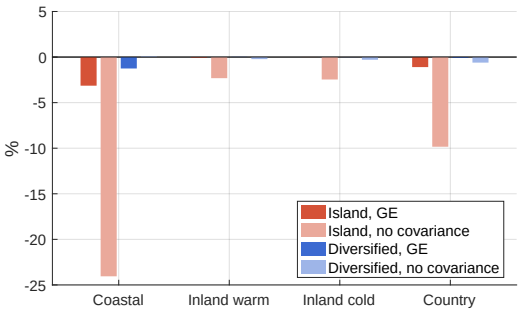
5.3 General Equilibrium and Diversification Provide Insurance

Our analysis uncovers two central forces that effectively insure the U.S. economy against climate risk: general equilibrium capital price adjustments, and diversification of investment. We illus-

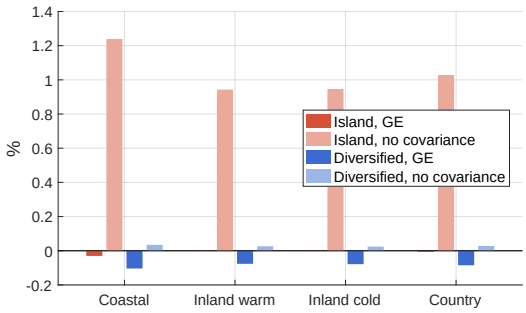
[10] Appendix Figures G2, G3, and G4 repeat the exercise in the diversified model.

Figure 8: Welfare cost of climate risk with and without the price covariance, by location group.

(a) Capitalist welfare / capital value (%)



(b) Worker welfare (%)



Note: Effect of all risk in 2100 along the 2°C transition: welfare with the trend and all risk minus welfare with the trend only, in percent of lifetime consumption. By group, panel (a) reports the welfare of island capitalists in the island model and the value of local capital in the diversified model. The country bars report aggregate capitalist welfare in both models. Panel (b): worker welfare, weighted by consumption and population. Island model in red, diversified model in blue. Dark bars: general equilibrium. Light bars ('no covariance'): valuation without the covariance between the price of capital and local shocks (Section 5.1). Groups as in Figure 7.

trate both in Figure 7, which reports the capital risk premium by location group.

In the island model, the risk premium without the price covariance is more than twice as large as in general equilibrium. To understand why, recall that investment is costly to reverse at the margin because of the adjustment cost. When a storm destroys part of the stock, the rental rate and the price of installed capital rise, and the increase in the price of the remaining stock partly hedges the loss. As a result, on the coast the premium is 1.5 p.p. per year without the price covariance and 0.6 p.p. with it. For the country as a whole these magnitudes are 0.4 and 0.2 p.p.

Once we allow investors to diversify, the capital risk premium falls to nearly zero with or without the price covariance. Any given location is too small to meaningfully affect the consumption of a diversified investor. This property is the consequence of three features of the data. First, no county is sufficiently large and exposed to be granular on its own. Second, climate shocks are only moderately correlated across space. Third, there are many alternative locations that are not exposed that provide useful hedges.

We find that the insurance from price responses removes a substantial fraction of the welfare cost of risk for island capitalists, and diversification removes most of what remains. Figure 8(a) reports the corresponding welfare cost of risk for capitalists. Since capitalist welfare is only defined for the country as a whole in the diversified model, we report the value of capital region by region, and capitalist welfare for the country as a whole.

Coastal capitalists in the island model lose 3.1% of lifetime consumption from risk in general equilibrium and 24% without the price covariance. For the country as a whole, these losses ag-

gregate to 1.1 and 10%. In the diversified model, the coastal value of capital falls by 1.3% and the diversified capitalist loses 0.1%. Without the price covariance, the coastal value of capital increases marginally: the price of capital is higher without the covariance, so that coastal investment and capital are higher than in general equilibrium. This increase is barely visible in Figure 8(a), but leads to a meaningful decline in consumption through the capitalist budget constraint.[11]

Figure 8(b) reports the welfare cost of risk for workers. In general equilibrium, workers lose a few hundredths of a percent on the coast in the island model, 0.007% for the country, and about 0.1% in the diversified model. Diversification hurts workers slightly: capital leaves risky locations hit by a storm faster when investment need not be self-financed and alternative investments exist, depressing real wages. Without the price covariance, island capitalists cannot rely on the price hedge and hold buffer stocks in their only asset: coastal capital rises by 17%, benefiting workers. As workers move in from inland locations, real wages rise inland, further benefiting workers there. Appendix Figure G5 shows the corresponding capital and population changes.

We corroborate our conclusions with a series of additional exercises. First, we highlight that the availability of profitable investment locations is key for diversification to lower the cost of risk. Appendix Figures G6 and G7 display the cost of risk in an economy in which we double the dispersion of local fundamentals across counties. In that economy, general equilibrium capital price adjustments are less beneficial, and capital risk premia rise also in the diversified economy.

Second, Appendix H.1 repeats the exercise with Epstein and Zin (1989) preferences with a unit elasticity of intertemporal substitution but the same risk aversion as in our baseline. We reach similar conclusions, with one exception: the precautionary motive of capitalists depends on the elasticity of intertemporal substitution rather than the risk aversion, so that separating both mutes worker welfare gains due to higher capital stocks.

5.4 Risk Is Less Costly Than the Trend

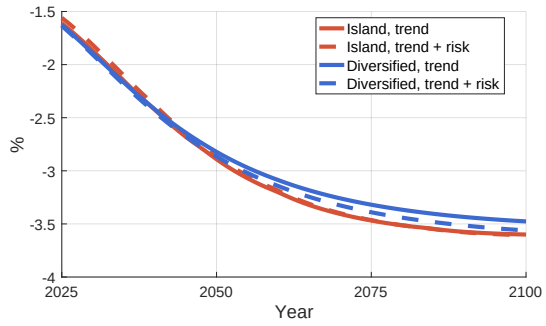
The moderate effects of risk, particularly in the diversified model, suggest that the deterministic trend in climate change might be more costly. Figure 9 compares the transition with the trend alone to the transition with local and aggregate risk.

Figure 9(a) shows aggregate worker welfare. By 2100 the trend costs workers 3.6% of lifetime consumption in the island model and 3.5% in the diversified model. With both local and aggregate risk, welfare losses are 3.6% in both models. In 2025, workers lose 1.6% in both models with trend

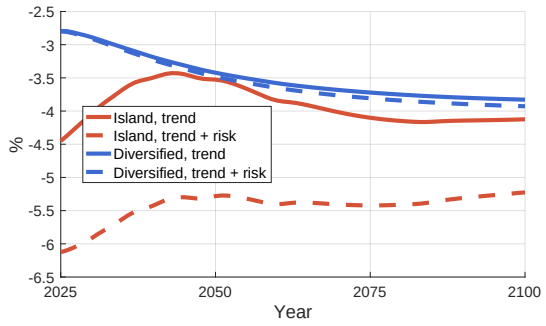
[11] Without the price covariance, markets can only clear if investors are off their optimality conditions on some margin, which here we take to be consumption.

Figure 9: Aggregate effects of warming with and without risk, 2025 to 2100.

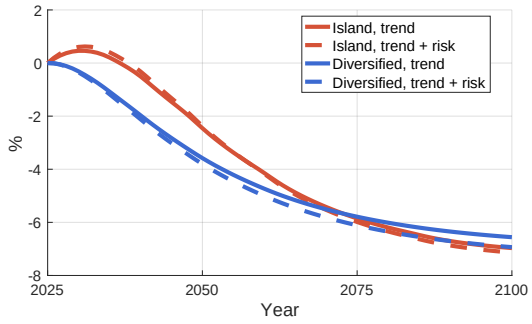
(a) Worker welfare



(b) Capitalist welfare



(c) Capital



Note: Transition to a 2°C warmer climate, 2025 to 2100. Island model in red, diversified model in blue. Solid lines: trend only. Dashed lines: trend with local and aggregate risk. Panel (a): aggregate worker welfare, the lifetime consumption equivalent at each date weighted by consumption and population, in percent. Panel (b): aggregate capitalist welfare, the consumption-weighted lifetime consumption equivalent of island capitalists and the diversified capitalist's equivalent, in percent. Panel (c): aggregate capital stock relative to 2025, in percent.

and risk. The effects of risk are small next to those of the trend because investors bear the brunt of risk.

Figure 9(b) shows aggregate capitalist welfare. The trend lowers the welfare of capitalists in the island model by 4.5% of lifetime consumption in 2025, and their welfare in 2100 stands at −4.1%. Risk adds 1.7 p.p. to the initial loss and takes the 2100 level to −5.2%. However, once diversification is allowed, the cost of risk falls to a tenth of a percentage point.[12]

Figure 9(c) shows the aggregate capital stock over time. It falls by approximately 7% regardless of diversification by 2100. Risk adds 0.2 and 0.4 p.p. to these losses in the island and diversified models, respectively. Interestingly, the capital stock in the diversified model falls faster at first, by

[12]The slight high-frequency non-convexities in the paths in Figure 9(b) follow from pasting solutions constructed around multiple expansion points for the deterministic trend. They do not affect the overall shape of the solution and are no larger than about a hundredth of a percentage point.

0.3% by 2030, while it rises slightly in the island model, as exposed owners save more when their prospects worsen. Capital leaves exposed places more rapidly when investment elsewhere need not be self-financed. By 2100 the capital stock in the island model is lower: more capital stays in exposed places in the medium run, where the trend raises depreciation, and capital rises less in safe locations, where investors must self-finance their expansion.

We assess the robustness of these conclusions with a range of additional exercises. First, we show that the moderate cost of risk is not due to aggregation. Appendix Figure G8 shows that it remains small even for specific counties. For instance, we find that Charleston loses 75 to 79% of its population and 89 to 92% of its capital by 2100 in both models, and risk changes these 2100 losses by about one percentage point. Second, these findings hold across a broad range of plausible values for risk aversion. Appendix Figures G9 and G10 vary these parameters. Unless capitalists are extremely risk-averse and cannot diversify across space at all, the cost of risk remains moderate for the U.S. economy. Third, these findings remain similar with Epstein and Zin (1989) preferences.

Our fourth exercise raises a relevant caveat to these conclusions. We explore how uninsurable within-county risk would affect the costs of risk. To do so, we assume that a fraction of capital owners receive larger capital depreciation shocks than the average, and others receive lower capital depreciation shocks than the average. Appendix Figure H3 shows that for plausible degrees of within-county risk, the costs of risk for capitalists in the island model can rise substantially. In the diversified model, where investors can diversify across and within locations, this mean-preserving spread within locations has no impact on allocations or welfare by definition.

6 Conclusion

A distinctive feature of climate change is that it exacerbates aggregate and local risk. On the one hand, we are uncertain about its aggregate severity and, therefore, about how it will affect specific regions. On the other hand, global warming will increase the variability of local weather patterns. These two forms of climate-induced risk are distinct and affect agents and regions differentially. The former, which we refer to as aggregate risk, involves persistent uncertainty about long-term climate paths, to which investors can react over time. The latter, which we refer to as local risk, is realized ex-post and destroys capital or lowers productivity in the location it hits. This distinction is essential for understanding the magnitude of the effects of each type of risk, its effect on capitalists and workers, and its impact on the distribution of economic activity across counties in the U.S.

We find that, in any given year, local risk from shifting weather distributions is far larger than aggregate risk from uncertain warming paths, although the persistence of aggregate risk makes their lifetime costs comparable for exposed investors. Still, equilibrium responses of capital prices, and diversification when it is available, insure investors against most of this risk. Hence, the additional cost from climate risk ends up being much lower than the cost of the warming trend itself, for both workers and capitalists.

Our paper is among the first to study the consequences of aggregate and local risk in a dynamic spatial economy with labor mobility and forward-looking capital investments. The model can be extended in several ways. Our two market structures bound the ability of investors to diversify. An intermediate structure, with a costly national bond or partial ownership of distant capital, would situate the economy between them. Furthermore, we treat a county as the unit at which storm damages are pooled. Of course, within a county, damages may fall unevenly on individual owners and may not be fully insurable. Our data do not allow us to measure this within-location dispersion, and we show in Appendix H.2 that it can raise the cost of risk for island capitalists substantially. Measuring uninsurable within-location risk from extreme weather is a promising avenue for future work. Finally, our framework can be used to study other forms of local and aggregate risk, beyond climate risk, and their consequences for local and economy-wide outcomes. As such, our paper offers a practical way of analyzing risk in dynamic spatial general equilibrium setups.

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A Microfoundation of the Depreciation and Productivity Processes

This appendix derives the depreciation and productivity processes of Section 2 from infrequent extreme events. The derivation gives the linear specification (5) and relates local volatilities to damage slopes.

A.1 Depreciation Shocks

We derive the depreciation process (2) and the linear specification (5) from infrequent, spatially correlated storms. A storm hitting location i destroys a fraction $D_i \in [0,1)$ of its capital stock. Storms arrive at a Poisson rate p_{it} that depends on global mean temperature, with correlated arrivals across locations. Over a short interval dt, the probability of a shock in i is $p_{it}\,dt$, while the probability of shocks in both i and $j \neq i$ is $p_{ij,t}\,dt$, where:

$$p_{ij,t} = \mathfrak{c}_{ij}\sqrt{p_{it}\,p_{jt}}, \qquad \mathfrak{c}_{ij} = \mathfrak{c}_{ji}.$$

Here $\mathfrak{c}_{ij}$ denotes the capital correlation $\mathfrak{c}_{\Delta,ij}$ in the main text. To see this, let $dJ_{it} \in \{0,1\}$ indicate a shock in i during dt. Then $\mathbb{E}[dJ_{it}\,dJ_{jt}] = p_{ij,t}\,dt$, while the product $\mathbb{E}[dJ_{it}]\,\mathbb{E}[dJ_{jt}]$ is of order dt^2. Since the indicators are binary, their variances are $p_{it}\,dt$ and $p_{jt}\,dt$ to first order. Their correlation therefore converges to $p_{ij,t}/\sqrt{p_{it}p_{jt}} = \mathfrak{c}_{ij}$ as $dt \to 0$.

The capital stock in location i follows a drift-compound Poisson process, $dK_{it}/K_{it} = g_{it}\,dt - d\overline{\Delta}_{it}$, with:

$$d\overline{\Delta}_{it} = \Delta_i\,dt + D_i\,dJ_{it},$$

where Δ_i denotes depreciation unrelated to extreme events. When a shock hits location i, its capital stock falls from K_{it} to $(1 - D_i)K_{it}$. For a function V of the vector of capital stocks $K = (K_1, \ldots, K_{\mathcal{I}})$, the generator associated with the jumps is the expected change in V per unit of time:

$$\mathcal{L}^{\text{jump}}[V](K) \equiv \lim_{dt\to 0} \frac{1}{dt}\,\mathbb{E}\Big[V\big((1 - D_1\,dJ_{1t})K_1, \ldots, (1 - D_{\mathcal{I}}\,dJ_{\mathcal{I}t})K_{\mathcal{I}}\big) - V(K)\Big]. \tag{A1}$$

We approximate this generator by expanding V to second order in the damage fractions D_i. Using $\mathbb{E}[dJ_{it}] = \mathbb{E}[dJ_{it}^2] = p_{it}\,dt$ and $\mathbb{E}[dJ_{it}\,dJ_{jt}] = p_{ij,t}\,dt$ for $j \neq i$, we obtain:

$$\begin{aligned}\mathcal{L}^{\text{jump}}[V](K) = &-\sum_i K_i \frac{\partial V}{\partial K_i}\,D_i\,p_{it} + \frac{1}{2}\sum_i K_i^2 \frac{\partial^2 V}{\partial K_i^2}\,D_i^2\,p_{it} \\ &+ \frac{1}{2}\sum_{i\neq j} K_i K_j \frac{\partial^2 V}{\partial K_i \partial K_j}\,D_i D_j\,\mathfrak{c}_{ij}\sqrt{p_{it}\,p_{jt}},\end{aligned}$$

up to second order. Holding the D_i fixed, this expression has the form of the generator of correlated Brownian motions, with one innovation per location. The mean destruction rate in i is $D_i p_{it}$, its variance rate is $D_i^2 p_{it}$, and the covariance rate between i and j is $D_i D_j \mathfrak{c}_{ij}\sqrt{p_{it}p_{jt}}$. We therefore approximate the jump process by the depreciation process (2) in the main text:

$$\begin{aligned} d\overline{\Delta}_{it} &= \Delta_{it}\,dt + \Sigma_{\Delta,it}\,dW_{\Delta,it}, \qquad \Delta_{it} = \Delta_i + D_i\,p_{it}, \\ \Sigma_{\Delta,it} &= D_i\sqrt{p_{it}}, \qquad \mathbb{E}_t[dW_{\Delta,it}dW_{\Delta,jt}] = \mathfrak{c}_{ij}\,dt, \end{aligned}$$

where the $dW_{\Delta,it}$ are standard Brownian increments with time-invariant correlations $\mathfrak{c}_{ij}$.

Motivated by the data, we assume storms arrive at rate $p_{it} = p_{i0} + p_{i1}(\overline{T}_t - T^B)$, where $p_{i0} \geq 0$ and $p_{i1} > 0$. We view these storms as the sum of two independent types: baseline storms, which arrive at rate p_{i0}, and storms added by warming, which arrive at rate $p_{i1}(\overline{T}_t - T^B)$. Each type is correlated across locations as above, with the same correlations $\mathfrak{c}_{ij}$. Because the two types are independent, a baseline storm in i and a storm added by warming in j coincide with a probability of order dt^2. The covariance rate between i and j is therefore $D_i D_j\,\mathfrak{c}_{ij}\big[\sqrt{p_{i0}p_{j0}} + \sqrt{p_{i1}p_{j1}}\,(\overline{T}_t - T^B)\big]$, while the mean and variance rates are unchanged. We absorb baseline expected storm destruction,

$D_i p_{i0}$, into Δ_i. Hence, writing $\delta_i \equiv D_i p_{i1}$, $\Sigma^2_{\Delta,i} \equiv D_i^2 p_{i1}$ and $\vartheta_{\Delta,i} \equiv p_{i0}/p_{i1}$, the depreciation process becomes:

$$\begin{aligned}
\Delta_{it} &= \Delta_i + \delta_i(\overline{T}_t - T^B), \\
\Sigma^2_{\Delta,it} &= \Sigma^2_{\Delta,i}\big(\vartheta_{\Delta,i} + \overline{T}_t - T^B\big), \\
\Sigma_{\Delta,ijt} &= \Sigma_{\Delta,i}\Sigma_{\Delta,j}\,\mathfrak{c}_{\Delta,ij}\big(\sqrt{\vartheta_{\Delta,i}\vartheta_{\Delta,j}} + \overline{T}_t - T^B\big).
\end{aligned} \tag{A2}$$

This is the specification in (5) when $\epsilon = 1$. Baseline storms add to the covariance the term $\Sigma_{\Delta,i}\Sigma_{\Delta,j}\mathfrak{c}_{\Delta,ij}\sqrt{\vartheta_{\Delta,i}\vartheta_{\Delta,j}}$, which does not depend on temperature. To second order, this term enters the solution separately, so that setting $\vartheta_{\Delta,i} = 0$, as in our quantification, isolates the effect of warming. The covariance is then $\Sigma_{\Delta,it}\Sigma_{\Delta,jt}\mathfrak{c}_{\Delta,ij}$, as in the main text. Moreover, with a common storm size D^{storm}, the small-damage approximation gives $\delta_i = D^{\text{storm}} p_i^{\text{storm}}$ and $\Sigma^2_{\Delta,i} = D^{\text{storm}}\delta_i$, where p_i^{storm} is the estimated change in storm probability per degree of warming.

A.2 Productivity Shocks

We derive the productivity process (1) from infrequent heat waves using the same argument. A heat wave in location i lowers log productivity χ_{it} by $D_i \geq 0$, where $Z_{it} = Z_i e^{\chi_{it}}$. Heat waves arrive at a Poisson rate p_{it}. Events in i and $j \neq i$ coincide over a short interval dt with probability $\mathfrak{c}_{ij}\sqrt{p_{it}p_{jt}}\,dt$. The damages, arrival rates and correlations now refer to heat waves. In particular, $\mathfrak{c}_{ij}$ denotes $\mathfrak{c}_{\chi,ij}$ in the main text and may differ from the storm correlation. Heat waves are independent of storms, so that productivity and depreciation shocks are uncorrelated.

Between events, productivity returns toward its baseline at rate $\kappa > 0$. The law of motion is $d\chi_{it} = -\kappa\chi_{it}\,dt - D_i\,dJ_{it}$. The generator acting on a function V of the productivity vector $\chi = (\chi_1, \ldots, \chi_{\mathcal{I}})$ is:

$$\mathcal{L}^{\text{jump}}[V](\chi) \equiv -\kappa\sum_i \chi_i \frac{\partial V}{\partial \chi_i} + \lim_{dt\to 0}\frac{1}{dt}\,\mathbb{E}\big[V(\chi_1 - D_1\,dJ_{1t}, \ldots, \chi_{\mathcal{I}} - D_{\mathcal{I}}\,dJ_{\mathcal{I}t}) - V(\chi)\big]. \tag{A3}$$

Expanding V to second order in the D_i gives a similar diffusion generator. In addition to the restoring drift $-\kappa\chi_i$, the jumps contribute a drift of $-D_i p_{it}$, a variance rate of $D_i^2 p_{it}$, and a covariance rate of $D_i D_j \mathfrak{c}_{ij}\sqrt{p_{it}p_{jt}}$ between locations i and j. The approximating diffusion has one innovation per location, $D_i\sqrt{p_{it}}\,dW_{\chi,it}$, with $\mathbb{E}_t[dW_{\chi,it}dW_{\chi,jt}] = \mathfrak{c}_{ij}\,dt$.

Similar to storms, we assume that the probability of a heat wave scales linearly with temperature, $p_{it} = p_{i0} + p_{i1}(\overline{T}_t - T^B)$, where $p_{i0} \geq 0$ and $p_{i1} > 0$. As for storms, baseline heat waves and

heat waves added by warming are independent, and each type is correlated across locations with $\mathfrak{c}_{\chi,ij}$. The drift is then $-\kappa\chi_{it} - D_i p_{i0} - D_i p_{i1}(\overline{T}_t - T^B)$, and the variance is $D_i^2 p_{i0} + D_i^2 p_{i1}(\overline{T}_t - T^B)$. The constant drift $-D_i p_{i0}$ shifts the baseline stationary mean of χ_{it} by $-D_i p_{i0}/\kappa$. We absorb this shift in Z_i and recenter χ_{it} around the new baseline. Writing $\kappa\chi_i \equiv -D_i p_{i1}$, $\Sigma_{\chi,i}^2 \equiv D_i^2 p_{i1}$ and $\vartheta_{\chi,i} \equiv p_{i0}/p_{i1}$ gives:

$$\begin{aligned} X_{it} &= \kappa\, \chi_i\, (\overline{T}_t - T^B), \\ \Sigma_{\chi,it}^2 &= \Sigma_{\chi,i}^2 \big(\vartheta_{\chi,i} + \overline{T}_t - T^B\big), \\ \Sigma_{\chi,ijt} &= \Sigma_{\chi,i}\Sigma_{\chi,j}\, \mathfrak{c}_{\chi,ij} \big(\sqrt{\vartheta_{\chi,i}\vartheta_{\chi,j}} + \overline{T}_t - T^B\big). \end{aligned} \tag{A4}$$

This is the productivity process (1), with the drift and variance of (5) when $\epsilon = 1$.

B Equilibrium Derivations

B.1 Static Equilibrium

Final good production. The wage w_{it} in location i satisfies the firm's first-order condition $w_{it} = (1-\alpha) Z_{it} (S_{it}/N_{it}^P)^{\alpha}$. Neither renters nor firms cover depreciation, so that the rental rate for structures satisfies $r_{it} = \alpha Z_{it} (S_{it}/N_{it}^P)^{-(1-\alpha)}$.

Buildings production. The optimal choice of labor in building production implies:

$$\varpi r_{it} B_{it}/N_{it}^B = w_{it}.$$

Housing demand. The local housing market clears: $\beta(1+s)\, w_{it} N_{it} = r_{it} H_{it}$.

Equilibrium prices. We solve for equilibrium prices and quantities as functions of the local capital stock K_{it} and local number of workers N_{it}. Combining labor demand in final good and building production, we obtain $\varpi\alpha \frac{B_{it}}{N_{it}^B} = (1-\alpha)\frac{S_{it}}{N_{it}^P}$. Housing demand can be written as $\beta(1+s)(1-\alpha) N_{it} \frac{S_{it}}{N_{it}^P} = \alpha H_{it}$.

We look for shares $\varkappa^N, \varkappa^H$ such that:

$$N_{it}^P = \varkappa^N N_{it} \;\;,\;\; H_{it} = \varkappa^H B_{it} \tag{B1}$$

and so $N_{it}^B = (1-\varkappa^N) N_{it}$ and $S_{it} = (1-\varkappa^H) B_{it}$. Substituting into the previous equations, we

obtain:

$$\frac{\varpi\alpha}{1-\alpha} = \frac{(1-\varkappa^H)(1-\varkappa^N)}{\varkappa^N} \quad , \quad \frac{\beta(1+s)(1-\alpha)}{\alpha} = \frac{\varkappa^N\varkappa^H}{1-\varkappa^H}.$$

These two equations imply:

$$\varkappa^N = \frac{(1-\alpha)(1-\varpi\beta(1+s))}{\alpha\varpi+(1-\alpha)} \quad , \quad \varkappa^H = \beta(1+s)\frac{(1-\alpha)+\alpha\varpi}{\alpha+(1-\alpha)\beta(1+s)}. \tag{B2}$$

Using these shares, we express the wage and the rental rate as:

$$\begin{aligned} w_{it} &= (1-\alpha)\alpha^\alpha\Xi^{-\alpha}Z_{it}(B_{it}/N_{it})^\alpha \\ r_{it} &= \alpha^\alpha\Xi^{1-\alpha}Z_{it}(B_{it}/N_{it})^{-(1-\alpha)} \end{aligned} \tag{B3}$$

where:

$$\Xi = (1-\alpha)\frac{\alpha+(1-\alpha)\beta(1+s)}{\alpha\varpi+(1-\alpha)}$$

Substituting the buildings production function gives prices in terms of capital:

$$\begin{aligned} w_{it} &= (1-\alpha)\alpha^\alpha\Xi^{-\alpha}(1-\varkappa^N)^{\varpi\alpha}Z_{it}L_i^{\omega\alpha}(K_{it}^{1-\omega-\varpi}N_{it}^{\varpi-1})^\alpha \\ r_{it} &= \alpha^\alpha\Xi^{1-\alpha}(1-\varkappa^N)^{-(1-\alpha)\varpi}Z_{it}L_i^{-\omega(1-\alpha)}(K_{it}^{1-\omega-\varpi}N_{it}^{\varpi-1})^{-(1-\alpha)} \end{aligned} \tag{B4}$$

Land rents. Land rents in location i are $\omega\, r_{it}B_{it}$. Housing demand and the first-order conditions of firms give $r_{it}B_{it} = r_{it}(H_{it}+S_{it}) = \left[\beta(1+s)+\alpha\varkappa^N/(1-\alpha)\right] w_{it}N_{it}$, so that land rents are the same multiple of the wage bill in every location and period, $\omega\, r_{it}B_{it} = s\, w_{it}N_{it}$, with $s = \omega\big(\alpha+(1-\alpha)\beta\big)/\big(1-\alpha+\alpha\varpi-\omega(1-\alpha)\beta\big)$. The national portfolio therefore pays a worker earning w_{it} the rebate $s\, w_{it}$.

Rental rate of capital. The rental rate of capital is:

$$\begin{aligned} R_{it} &= (1-\omega-\varpi)(1-\varkappa^N)^\varpi K_{it}^{-(\omega+\varpi)}L_i^\omega N_{it}^\varpi r_{it} \\ &= (1-\omega-\varpi)\alpha^\alpha\Xi^{1-\alpha}(1-\varkappa^N)^{\alpha\varpi}Z_{it}L_i^{\omega\alpha}K_{it}^{-\phi}N_{it}^{1-\alpha+\alpha\varpi} \end{aligned} \tag{B5}$$

Writing productivity as its baseline times its time-varying component gives:

$$\begin{aligned} R_{it} &= \underbrace{(1-\omega-\varpi)\alpha^{\alpha}\Xi^{1-\alpha}(1-\varkappa^{N})^{\alpha\varpi}Z_{i}L_{i}^{\omega\alpha}}_{\equiv R_{0i}} e^{\chi_{it}}K_{it}^{-\phi}N_{it}^{\psi} \\ &= R_{0i}e^{\chi_{it}}K_{it}^{-\phi}N_{it}^{\psi} \\ &\equiv R_{i}(\chi_{it},K_{it},N_{it}) \end{aligned} \tag{B6}$$

where $\phi = \omega + \varpi + (1-\omega-\varpi)(1-\alpha)$ and $\psi = 1-\alpha+\alpha\varpi$.

Consumption and flow utility. Total consumption (real income) in location i is:

$$\begin{aligned} c_{it} &= (1+s)\frac{w_{it}}{r_{it}^{\beta}} \\ &= (1+s)\frac{(1-\alpha)\alpha^{\alpha}\Xi^{-\alpha}(1-\varkappa^{N})^{\varpi\alpha}Z_{it}L_{i}^{\omega\alpha}(K_{it}^{1-\omega-\varpi}N_{it}^{\varpi-1})^{\alpha}}{\left[\alpha^{\alpha}\Xi^{1-\alpha}(1-\varkappa^{N})^{-(1-\alpha)\varpi}Z_{it}L_{i}^{-\omega(1-\alpha)}(K_{it}^{1-\omega-\varpi}N_{it}^{\varpi-1})^{-(1-\alpha)}\right]^{\beta}} \\ &= (1+s)(1-\alpha)\alpha^{\alpha(1-\beta)}\Xi^{-\xi}(1-\varkappa^{N})^{\varpi\xi}Z_{it}^{1-\beta}L_{i}^{\omega\xi}K_{it}^{(1-\omega-\varpi)\xi}N_{it}^{-\xi(1-\varpi)} \\ &= \underbrace{(1+s)(1-\alpha)\alpha^{\alpha(1-\beta)}\Xi^{-\xi}(1-\varkappa^{N})^{\varpi\xi}Z_{i}^{1-\beta}L_{i}^{\omega\xi}}_{\equiv c_{0i}^{w}} \cdot e^{(1-\beta)\chi_{it}}\left(\frac{K_{it}^{1-\omega-\varpi}}{N_{it}^{1-\varpi}}\right)^{\xi} \end{aligned} \tag{B7}$$

where $\xi = \alpha + \beta(1-\alpha)$. To obtain flow utility in location i at time t, we use amenities $A_{it} = A_{i}e^{a_{it}}$ with $a_{it} = \eta\chi_{it}$. The consumption index of the worker is $\widetilde{C}_{it} \equiv A_{it}(1+s)\, w_{it}/r_{it}^{\beta}$, so that:

$$u\left(A_{it}(1+s)\frac{w_{it}}{r_{it}^{\beta}}\right) = u\left(C_{0i}^{w}e^{a_{it}+(1-\beta)\chi_{it}}\left(\frac{K_{it}^{1-\omega-\varpi}}{N_{it}^{1-\varpi}}\right)^{\xi}\right) \equiv U_{i}(a_{it},\chi_{it},K_{it},N_{it}), \tag{B8}$$

where $C_{0i}^{w} \equiv A_{i}c_{0i}^{w}$.

B.2 Proof of Proposition 1

We prove Proposition 1 by solving the diversified capitalist's problem using a stochastic maximum principle.

The capitalist chooses consumption C_t, the investment rates g_{it} and the bond position B_t, taking the rental rates and the bond rate as given. Capital in location i follows the level form of (4), and

the bond position follows the budget constraint (9) in differential form:

$$\frac{dK_{it}}{K_{it}} = \big(g_{it} - \Delta_{it}\big)\, dt - \Sigma_{\Delta,it}\, dW_{\Delta,it}, \tag{B9}$$

$$dB_t = \Big[\sum_i R_{it} K_{it} + \overline{R}_t B_t - C_t - \sum_i c_i(g_{it}) K_{it}\Big]\, dt, \tag{B10}$$

where the adjustment costs $c_i(\cdot)$ are given by (3). The sequential problem is:

$$\max_{\{C_t, \{g_{it}\}_i\}_{t\geq 0}} \mathbb{E}_0 \left[\int_0^\infty e^{-\rho t}\, \overline{u}(C_t)\, dt\right], \tag{B11}$$

subject to (B9), (B10) and the usual no-Ponzi and transversality conditions.

Let λ_t be the current-value costate on bond wealth and λ_{it} the current-value costate on capital in location i. Because capital is risky, the current-value Hamiltonian includes a quadratic-covariation term. This is the expectation form of the diffusion term in the stochastic maximum principle:

$$\begin{aligned} \mathcal{H}_t \;=\; & \overline{u}(C_t) + \lambda_t \Big[\sum_i R_{it} K_{it} + \overline{R}_t B_t - C_t - \sum_i c_i(g_{it}) K_{it}\Big] \\ & + \sum_i \lambda_{it} \big(g_{it} - \Delta_{it}\big) K_{it} + \sum_i \frac{1}{dt}\, \mathbb{E}_t\big[d\lambda_{it}\, dK_{it}\big]. \end{aligned} \tag{B12}$$

The first-order conditions for consumption and investment are:

$$\lambda_t = \overline{u}'(C_t), \qquad\qquad \lambda_{it} = \lambda_t\, c_i'(g_{it}). \tag{B13}$$

The costate equation for bond wealth is given by:

$$\frac{1}{dt}\, \mathbb{E}_t[d\lambda_t] = \rho \lambda_t - \frac{\partial \mathcal{H}_t}{\partial B_t} = (\rho - \overline{R}_t)\, \lambda_t. \tag{B14}$$

Similarly, the costate equation for capital in i is

$$\frac{1}{dt}\, \mathbb{E}_t\Big[d\lambda_{it} + d\lambda_{it}\, \frac{dK_{it}}{K_{it}}\Big] = \rho \lambda_{it} - \lambda_t \big[R_{it} - c_i(g_{it})\big] - \lambda_{it}\big(g_{it} - \Delta_{it}\big), \tag{B15}$$

where the last term of (B12) adds an extra covariation term. Equations (B14) and (B15) are the bond and capital Euler equations in costate form.

Now, define the stochastic discount factor using the capitalist's marginal utility, and the price

of capital Q_{it} as the shadow value of installed capital in units of the final good:

$$\Lambda_t \equiv e^{-\rho t}\,\frac{\overline{u}'(C_t)}{\overline{u}'(C_0)}, \qquad\qquad Q_{it} \equiv \frac{\lambda_{it}}{\lambda_t}. \tag{B16}$$

The investment condition (B13) then gives the investment rule of the proposition:

$$Q_{it} = c_i'(g_{it}) = c_{i0}^{-1/\zeta}\, g_{it}^{1/\zeta} \qquad \Longleftrightarrow \qquad g_{it} = c_{i0}\, Q_{it}^{\zeta}. \tag{B17}$$

By (B13), $d\Lambda_t/\Lambda_t = d\lambda_t/\lambda_t - \rho\, dt$. The bond adjoint equation (B14) implies that a marginal bond position, which compounds at $\overline{R}_t$ and has no diffusion, satisfies $\mathbb{E}_t[d(\Lambda_t B_t)] = 0$. Equivalently:

$$\overline{R}_t = -\frac{1}{\Lambda_t}\,\mathbb{E}_t\left[\frac{d\Lambda_t}{dt}\right]. \tag{B18}$$

Substituting $\lambda_{it} = \lambda_t Q_{it}$ into (B15), dividing by $\Lambda_t K_{it}$ and applying Itô's product rule gives $0 = \frac{1}{\Lambda_t K_{it}}\mathbb{E}_t[d(\Lambda_t Q_{it} K_{it})]/dt + R_{it} - c_i(g_{it})$. Using $c_i(g_{it}) = \frac{\zeta c_{i0}}{1+\zeta} Q_{it}^{1+\zeta}$ from (B17), we can write this equation in Bellman form:

$$\begin{aligned} \overline{R}_t Q_{it} \;&=\; R_{it} + \frac{c_{i0}}{1+\zeta}\, Q_{it}^{1+\zeta} - \Delta_{it} Q_{it} + \mathbb{E}_t\left[\frac{dQ_{it}}{dt}\right] \\ &\quad + \frac{Q_{it}}{dt}\left\{\mathbb{C}\mathrm{ov}_t\left[\frac{d\Lambda_t}{\Lambda_t}, \frac{dQ_{it}}{Q_{it}}\right] + \mathbb{C}\mathrm{ov}_t\left[\frac{d\Lambda_t}{\Lambda_t} + \frac{dQ_{it}}{Q_{it}}, \frac{dK_{it}}{K_{it}}\right]\right\}, \end{aligned} \tag{B19}$$

where $\mathbb{C}\mathrm{ov}_t$ is the conditional quadratic covariation over dt.

We now characterize capitalist consumption as a diffusion process and express the stochastic discount factor and the risk-free rate in terms of its loadings.

All capitalists are identical and the bond is in zero net supply, so bond market clearing imposes $B_t = 0$. Setting $B_t = dB_t = 0$ in (B10) and using the rental rate (B6) and the investment rule (B17) gives:

$$\begin{aligned} C_t \;&=\; \sum_i \left[R_{it} - c_i(g_{it})\right] K_{it} = \sum_i \left[R_{0i}\, e^{\chi_{it}} K_{it}^{1-\phi} N_{it}^{\psi} - \frac{\zeta c_{i0}}{1+\zeta}\, K_{it} Q_{it}^{1+\zeta}\right] \\ &\equiv\; \sum_i \mathcal{C}_i(\chi_{it}, K_{it}, N_{it}, Q_{it}), \end{aligned} \tag{B20}$$

which is the consumption expression of the proposition. Write the growth rate of capitalist consumption as:

$$\frac{dC_t}{C_t} = \mu_t^C\, dt + \sigma_{z,t}^C\, dW_t + \sum_\ell \sigma_{\chi,\ell,t}^C\, dW_{\chi,\ell t} + \sum_\ell \sigma_{\Delta,\ell,t}^C\, dW_{\Delta,\ell t},$$

and denote by $\|\sigma_t^C\|^2 \equiv (\sigma_{z,t}^C)^2 + \sum_{\ell,m} \mathfrak{c}_{\chi,\ell m}\, \sigma_{\chi,\ell,t}^C \sigma_{\chi,m,t}^C + \sum_{\ell,m} \mathfrak{c}_{\Delta,\ell m}\, \sigma_{\Delta,\ell,t}^C \sigma_{\Delta,m,t}^C$ its variance rate, $\frac{1}{dt}\mathbb{V}\mathrm{ar}_t(dC_t/C_t)$. The correlations of the increments enter through $\mathfrak{c}_{\chi,\ell m}$ and $\mathfrak{c}_{\Delta,\ell m}$. With $\overline{u}'(C) = C^{-\overline{\gamma}}$, Itô's lemma applied to (B16) gives a drift $-\rho - \overline{\gamma}\,\mu_t^C + \frac{\overline{\gamma}(\overline{\gamma}+1)}{2}\|\sigma_t^C\|^2$ and a loading $-\overline{\gamma}\sigma^C$ on every increment. The bond Euler equation (B18) requires the drift to equal $-\overline{R}_t$. The stochastic discount factor is therefore given by

$$\frac{d\Lambda_t}{\Lambda_t} = -\overline{R}_t\, dt - \overline{\gamma}\,\sigma_{z,t}^C\, dW_t - \overline{\gamma}\sum_\ell \sigma_{\chi,\ell,t}^C\, dW_{\chi,\ell t} - \overline{\gamma}\sum_\ell \sigma_{\Delta,\ell,t}^C\, dW_{\Delta,\ell t}, \tag{B21}$$

where the three families of loadings correspond to the stochastic components of capitalist consumption growth, $dC_t/C_t - \mu_t^C\, dt$. The corresponding risk-free rate is

$$\overline{R}_t = \rho + \overline{\gamma}\,\mu_t^C - \frac{\overline{\gamma}(\overline{\gamma}+1)}{2}\,\|\sigma_t^C\|^2, \tag{B22}$$

which is the risk-free rate of the proposition, since $\mu_t^C = \frac{1}{dt}\mathbb{E}_t[dC_t/C_t]$ and $\|\sigma_t^C\|^2 = \frac{1}{dt}\mathbb{V}\mathrm{ar}_t(dC_t/C_t)$.

It remains to evaluate the covariance terms in (B19). By (B21), the innovation of the stochastic discount factor is $-\overline{\gamma}$ times the innovation of consumption growth, $d\Lambda_t/\Lambda_t + \overline{R}_t\, dt = -\overline{\gamma}\big(dC_t/C_t - \mu_t^C\, dt\big)$. Hence:

$$\begin{aligned}
\mathbb{C}\mathrm{ov}_t\left[\frac{d\Lambda_t}{\Lambda_t}, \frac{dQ_{it}}{Q_{it}}\right] &= -\overline{\gamma}\,\mathbb{C}\mathrm{ov}_t\left[\frac{dC_t}{C_t}, \frac{dQ_{it}}{Q_{it}}\right], \\
\mathbb{C}\mathrm{ov}_t\left[\frac{d\Lambda_t}{\Lambda_t} + \frac{dQ_{it}}{Q_{it}}, \frac{dK_{it}}{K_{it}}\right] &= -\overline{\gamma}\,\mathbb{C}\mathrm{ov}_t\left[\frac{dC_t}{C_t}, \frac{dK_{it}}{K_{it}}\right] + \mathbb{C}\mathrm{ov}_t\left[\frac{dQ_{it}}{Q_{it}}, \frac{dK_{it}}{K_{it}}\right].
\end{aligned}$$

Substituting into (B19) and dividing by Q_{it} gives:

$$\begin{aligned}
0 &= \frac{R_{it}}{Q_{it}} - \overline{R}_t + \frac{c_{i0}}{1+\zeta}\, Q_{it}^{\zeta} - \Delta_{it} + \mathbb{E}_t\left[\frac{1}{Q_{it}}\frac{dQ_{it}}{dt}\right] \\
&\quad -\overline{\gamma}\,\frac{1}{dt}\mathbb{C}\mathrm{ov}_t\left[\frac{dK_{it}}{K_{it}} + \frac{dQ_{it}}{Q_{it}}, \frac{dC_t}{C_t}\right] + \frac{1}{dt}\mathbb{C}\mathrm{ov}_t\left[\frac{dQ_{it}}{Q_{it}}, \frac{dK_{it}}{K_{it}}\right].
\end{aligned} \tag{B23}$$

The remaining covariance is the hedge. The price of capital is a function of the aggregate state, $Q_{it} = Q_i(z_t, \chi_t, N_t, \mathcal{K}_t, t)$. By (4), log capital in location ℓ has diffusion $-\Sigma_{\Delta,\ell t}\, dW_{\Delta,\ell t}$, so the loading of dQ_{it} on $dW_{\Delta,\ell t}$ is $-\Sigma_{\Delta,\ell t}\,\partial Q_{it}/\partial \mathcal{K}_\ell$. Since the diffusion of dK_{it}/K_{it} is $-\Sigma_{\Delta,it}\, dW_{\Delta,it}$ by (B9), we have:

$$\frac{1}{dt}\,\mathbb{C}\mathrm{ov}_t\Big(dQ_{it}, \frac{dK_{it}}{K_{it}}\Big) = \sum_\ell \Sigma_{\Delta,it}\Sigma_{\Delta,\ell t}\,\mathfrak{c}_{\Delta,i\ell}\,\frac{\partial Q_{it}}{\partial \mathcal{K}_\ell} = \sum_\ell \Sigma_{\Delta,i\ell t}\,\frac{\partial Q_{it}}{\partial \mathcal{K}_\ell}, \tag{B24}$$

using $\Sigma_{\Delta,i\ell t} = \Sigma_{\Delta,it}\Sigma_{\Delta,\ell t}\,\mathfrak{c}_{\Delta,i\ell}$. Substituting (B24) into (B23) gives the pricing equation of the proposition. Together with the investment rule (B17), aggregate consumption (B20) and the risk-free rate (B22), this completes the proof.

B.3 Proof of Proposition 2

We prove Proposition 2 by guessing and verifying the value of an island capitalist. The island problem is homogeneous of degree $1-\overline{\gamma}$ in capital. We therefore conjecture a value function of the form:

$$\mathcal{Q}_{it}(K) = \frac{(Q_{it}K)^{1-\overline{\gamma}}}{1-\overline{\gamma}} + \overline{Q}, \qquad \overline{Q} = -\frac{1}{\rho(1-\overline{\gamma})}. \tag{B25}$$

The constant $\overline{Q}$ absorbs the constant $-1/(1-\overline{\gamma})$ of the flow utility $\overline{u}$, since $\rho\overline{Q} = -1/(1-\overline{\gamma})$. Substituting the guess into the Bellman equation (11) and collecting the terms proportional to $K^{1-\overline{\gamma}}$, we obtain:

$$\begin{aligned}\frac{\rho\, Q_{it}^{1-\overline{\gamma}}}{1-\overline{\gamma}} &= \max_{g}\left\{\frac{[R_{it}-c_i(g)]^{1-\overline{\gamma}}}{1-\overline{\gamma}} + gQ_{it}^{1-\overline{\gamma}}\right\} - \left[\Delta_{it} + \frac{\overline{\gamma}\,\Sigma_{\Delta,it}^2}{2}\right] Q_{it}^{1-\overline{\gamma}} \\ &\quad + (1-\overline{\gamma})\, Q_{it}^{-\overline{\gamma}} \sum_j \Sigma_{\Delta,ijt}\frac{\partial Q_{it}}{\partial \mathcal{K}_j} + \frac{1}{1-\overline{\gamma}}\,\mathbb{E}_t\left[\frac{dQ_{it}^{1-\overline{\gamma}}}{dt}\right].\end{aligned} \tag{B26}$$

The first-order condition is:

$$c_i'(g)\left[R_{it}-c_i(g)\right]^{-\overline{\gamma}} = Q_{it}^{1-\overline{\gamma}} \quad\Longleftrightarrow\quad c_i'(g_{it}) = Q_{it}^{1-\overline{\gamma}}\left(\frac{\overline{C}_{it}}{K_{it}}\right)^{\overline{\gamma}} = \frac{\partial \mathcal{Q}_{it}/\partial K}{\overline{u}'(\overline{C}_{it})}, \tag{B27}$$

which is the investment rule of the proposition. Its solution is the investment rate $g_i(Q_{it}, R_{it})$.

Now, by Itô's lemma, $\mathbb{E}_t[dQ_{it}^{1-\overline{\gamma}}] = (1-\overline{\gamma})\, Q_{it}^{-\overline{\gamma}}\big(\mathbb{E}_t[dQ_{it}] - \frac{\overline{\gamma}}{2}\,\mathbb{V}\mathrm{ar}_t[dQ_{it}]/Q_{it}\big)$. Substituting this expression into (B26) and dividing by $Q_{it}^{1-\overline{\gamma}}$ gives:

$$\begin{aligned}\frac{\rho}{1-\overline{\gamma}} &= Q_{it}^{\overline{\gamma}-1}\max_{g}\left\{\frac{[R_{it}-c_i(g)]^{1-\overline{\gamma}}}{1-\overline{\gamma}} + gQ_{it}^{1-\overline{\gamma}}\right\} - \Delta_{it} - \frac{\overline{\gamma}}{2}\left\{\Sigma_{\Delta,it}^2 + \frac{1}{dt}\mathbb{V}\mathrm{ar}_t\left[\frac{dQ_{it}}{Q_{it}}\right]\right\} \\ &\quad + (1-\overline{\gamma})\sum_j \Sigma_{\Delta,ijt}\frac{1}{Q_{it}}\frac{\partial Q_{it}}{\partial \mathcal{K}_j} + \mathbb{E}_t\left[\frac{1}{Q_{it}}\frac{dQ_{it}}{dt}\right].\end{aligned} \tag{B28}$$

At the optimum, the first-order condition (B27) gives

$$Q_{it}^{\overline{\gamma}-1}[R_{it}-c_i(g_{it})]^{1-\overline{\gamma}} = [R_{it}-c_i(g_{it})]/c_i'(g_{it}),$$

so that the first term on the right-hand side equals $\frac{1}{1-\overline{\gamma}} \frac{R_{it} - c_i(g_{it})}{c_i'(g_{it})} + g_{it}$. Moving $\rho/(1-\overline{\gamma})$ to the right-hand side completes the proof.

B.4 Steady State Computation

We first derive the deterministic steady state of the diversified economy and show that the island economy delivers the same allocation, despite differences in the valuation of capital. We then describe a simple algorithm for computing this steady state.

At baseline temperatures, we set $\epsilon = 0$, so productivity is at its mean $\chi = 0$ and all volatilities vanish, which removes the covariance and risk terms from the pricing equations. Time derivatives also vanish in steady state. In both market structures, workers' values and the population distribution satisfy equations (12) and (13), respectively.

In the diversified economy, capitalist consumption is constant in steady state, so the bond Euler equation implies that the risk-free rate equals ρ. Combining the investment rule, the capital law of motion (4), and the pricing equation in Proposition 1, we obtain:

$$c_{i0}(Q_i^{ss})^{\zeta} = \Delta_i, \qquad (\rho + \Delta_i) Q_i^{ss} = R_i^{ss} + \frac{c_{i0}}{1+\zeta} (Q_i^{ss})^{1+\zeta}.$$

Investment in steady state implies $Q_i^{ss} = (\Delta_i / c_{i0})^{1/\zeta} = c_i'(\Delta_i)$. Substituting into the second condition yields:

$$R_i^{ss} = c_i(\Delta_i) + \rho c_i'(\Delta_i).$$

The island economy delivers the same deterministic steady-state allocation. The main difference is the interpretation of Q_i, which is proportional to certainty-equivalent consumption per unit of capital rather than the price of installed capital. In this economy, the capital law of motion (4) and the value equation in Proposition 2, in which the risk and covariance terms vanish and Q_i is constant, imply:

$$g_i(Q_i^{ss}, R_i^{ss}) = \Delta_i, \qquad \frac{R_i^{ss} - c_i(\Delta_i)}{c_i'(\Delta_i)} = \rho.$$

The second condition gives the same rental rate as in the diversified economy:

$$R_i^{ss} = c_i(\Delta_i) + \rho c_i'(\Delta_i).$$

The investment rule of Proposition 2 then gives $(Q_i^{ss})^{1-\overline{\gamma}} = c_i'(\Delta_i)[R_i^{ss} - c_i(\Delta_i)]^{-\overline{\gamma}} = \rho^{-\overline{\gamma}} c_i'(\Delta_i)^{1-\overline{\gamma}}$, so that the value of capital in the island economy is $Q_i^{ss} = \rho^{\frac{\overline{\gamma}}{\overline{\gamma}-1}} c_i'(\Delta_i)$. Thus, although the valuation of capital differs across market structures, both economies share the same steady-state investment rates, capital stocks, and population distribution.

These conditions provide a common procedure for computing the steady state. Given the depreciation rates Δ_i and investment cost parameters c_{i0}, we first recover the rental rates R_i^{ss} and the price of installed capital $c_i'(\Delta_i)$. We then solve for the remaining equilibrium variables as follows:

1. Use the rental-rate equation (B6) to express capital as a function of population:

$$K_i^{ss} = \left(\frac{R_{0i}(N_i^{ss})^{\psi}}{R_i^{ss}} \right)^{1/\phi} \equiv \overline{K}_i(N_i^{ss}). \tag{B29}$$

2. Substitute (B29) into the workers' value equation (12) to obtain:

$$\rho V_i^{ss} = U_i\big(0, 0, \overline{K}_i(N_i^{ss}), N_i^{ss}\big) + \mathcal{M}_i[V^{ss}]. \tag{B30}$$

3. Solve for equilibrium values V^{ss} and population N^{ss} by alternating between (B30) and (13):

   (a) Start from an initial guess for N^{ss}.

   (b) Given N^{ss}, solve (B30) for V^{ss}.

   (c) Given V^{ss}, update N^{ss} using (13).

   (d) Repeat steps (b) and (c) until both N^{ss} and V^{ss} converge.

C Perturbation Solution

This appendix proves Propositions 4 and 5. We first introduce the tensor notation, expansion conventions, and auxiliary lemmas in Appendix C.1, and express the Master Equations of Proposition 3 in state space in Appendix C.2. We then derive the FAME and SAME in Appendices C.3 and C.4, respectively. We focus on the diversified economy and present the modifications for the island economy at the end of Appendices C.3.2 and C.4.2. Finally, Appendix C.5 provides the second-order expansions of prices and investment used in estimation.

C.1 Tensor Notation and Auxiliary Lemmas

We begin by introducing the tensor operations and expansion conventions used below, together with the auxiliary lemmas needed for the derivations.

Let $\mathbb{X}$ be a tensor with entries $\mathbb{X}_{ijh}$, A a matrix, and a a vector. We define the three tensor products as:

$$\mathbb{X} \times_1 A \equiv \Big[\sum_i \mathbb{X}_{ijh} A_{\ell i}\Big]_{\ell jh}, \quad \mathbb{X} \times_2 A \equiv \Big[\sum_j \mathbb{X}_{ijh} A_{j\ell}\Big]_{i\ell h}, \quad \mathbb{X} \times_3 A \equiv \Big[\sum_h \mathbb{X}_{ijh} A_{h\ell}\Big]_{ij\ell}.$$

As in matrix multiplication, $\times_1$ contracts over the columns of A, while $\times_2$ and $\times_3$ contract over its rows. For vectors, we contract along indices 2 and 3 without transposition: $\mathbb{X} \times_2 a \equiv [\sum_j \mathbb{X}_{ijh} a_j]_{ih}$ and $\mathbb{X} \times_3 a \equiv [\sum_h \mathbb{X}_{ijh} a_h]_{ij}$.

We also define the contraction over the last two indices as $\mathbb{X} \times_{2,3} A \equiv \mathbf{vec}\big(\sum_{j,h} \mathbb{X}_{ijh} A_{jh}\big)$. This operation returns an $\mathcal{I}$-dimensional vector and is used to contract a Hessian with a covariance matrix. For a vector a, the diagonal tensor $\mathbf{diag}_3(a)$ has entries $\mathbf{diag}_3(a)_{iii} = a_i$ and zero elsewhere, implying $\mathbf{diag}_3(a) \times_2 b = \mathbf{diag}_3(a) \times_3 b = \mathbf{diag}(a \cdot b)$.

Finally, $a \cdot b$ denotes the componentwise product of two vectors, and $\mathbb{X}^*$ the transpose of the second and third indices, with $\mathbb{X}^*_{ijh} \equiv \mathbb{X}_{ihj}$. For a matrix or a vector, A^* is the ordinary transpose. We use $\mathbb{1}$ for the vector of ones and $\mathbb{1}_{x=n}$ for the indicator function.

Conventions for the expansion. We expand the Master Equations in ϵ around the deterministic steady state of the final, 2°C warmer climate, computed as in Appendix B.4, using the scaled deviations n, k, and $\widehat{\chi}$ defined in Section 3. In the transition, each year uses the expansion around the steady state whose climate is closest to its own (Section 5.1), and the derivations below apply to each expansion point. Following (16) and (17), we write $V_t = V^{ss} + \epsilon\, v_{1t} + \frac{\epsilon^2}{2} v_{2t}$, with $v_{1t} = \sum_x V^x x + v_t^z z + v_t^T$, and similarly for Q_t. The tensor terms of v_{2t} take the form $\mathcal{V}^{x,y} \times_3 y \times_2 x$, whose ith entry is $\sum_{j,h} \mathcal{V}^{x,y}_{ijh} x_j y_h$. As in Section 3, $x, y \in \{n, k, \chi\}$ index the three distributions and also denote their corresponding deviation vectors n, k, and $\widehat{\chi}$. The sums $\sum_x$ and $\sum_s$ run over these distributions, while $\sum_{x,y}$ runs over the nine ordered pairs. We retain the original state labels $\mathcal{K}$ and N in the subscripts of flow matrices, so that $\mathrm{D}_{U\mathcal{K}} = \mathrm{D}_{Uk}$ and $\mathrm{D}_{rN} = \mathrm{D}_{rn}$. We denote the stacked state deviation by $\mathbf{s}_t \equiv (n_t, k_t, \widehat{\chi}_t)$. Throughout, lowercase letters denote vectors, uppercase letters matrices, calligraphic letters the unknown tensors $\mathcal{V}^{x,y}$ and $\mathcal{Q}^{x,y}$, and double-struck letters known tensors, such as the outer products $\mathbb{V}^{x,y}$ and $\mathbb{Q}^{x,y}$ of Lemma 2 with $W = V, Q$.

Since (16) identifies only the sum $\mathcal{V}^{x,y}_{ijh} + \mathcal{V}^{y,x}_{ihj}$, we symmetrize the coefficients throughout. A

term $\mathbb{F} \times_3 y \times_2 x$ contributes $\frac{1}{2}\mathbb{F}$ to the pair (x, y) and $\frac{1}{2}\mathbb{F}^*$ to the pair (y, x). Under this convention, the equations for (x, y) and (y, x) are transposes of each other, and the diagonal tensors $\mathcal{V}^{x,x}$ are symmetric in their second and third indices.

Auxiliary lemmas. The first lemma collects three identities that follow by expanding the products.

Lemma 1. *For a tensor $\mathbb{T}$, matrices A, B and vectors a, b:*

(i) $\mathbb{T} \times_3 (a+b) \times_2 (a+b) = \mathbb{T} \times_3 a \times_2 a + (\mathbb{T} + \mathbb{T}^*) \times_3 a \times_2 b + \mathbb{T} \times_3 b \times_2 b$. *If $\mathbb{T}$ is symmetric in indices 2 and 3, then $\mathbb{T} \times_3 a \times_2 b = \mathbb{T} \times_3 b \times_2 a = \mathbb{T} \times_2 a \times_3 b$ and the middle term is $2\,\mathbb{T} \times_3 a \times_2 b$.*

(ii) $A\big(\mathbb{T} \times_3 a \times_2 b\big) = \big(\mathbb{T} \times_1 A\big) \times_3 a \times_2 b$.

(iii) $\mathbb{T} \times_3 (Aa) \times_2 (Bb) = \mathbb{T} \times_3 A \times_2 B \times_3 a \times_2 b$.

The second lemma expands the quadratic terms in the first-order solution.

Lemma 2. *Let $w_{1t} = \sum_x W^x\, x + w_t^z\, z + w_t^T$, and let $\mathbb{T}$ be symmetric in its last two indices. Then:*

$$\begin{aligned}
\mathbb{T} \times_3 w_{1t} \times_2 w_{1t} &= \sum_{x,y} \big(\mathbb{T} \times_3 W^y \times_2 W^x\big) \times_3 y \times_2 x + 2\sum_x \big(\mathbb{T} \times_2 W^x \times_3 w_t^z\big)\, x\, z \\
&\quad + 2\sum_x \big(\mathbb{T} \times_2 W^x \times_3 w_t^T\big)\, x + \big(\mathbb{T} \times_3 w_t^z \times_2 w_t^z\big) z^2 \\
&\quad + \big(\mathbb{T} \times_3 w_t^T \times_2 w_t^T\big) + 2\big(\mathbb{T} \times_3 w_t^z \times_2 w_t^T\big) z, \\
w_{1t} \cdot w_{1t} &= \sum_{x,y} \mathbb{W}^{x,y} \times_3 y \times_2 x + 2\sum_x \boldsymbol{diag}(w_t^z)\, W^x\, x\, z + 2\sum_x \boldsymbol{diag}(w_t^T)\, W^x\, x \\
&\quad + (w_t^z \cdot w_t^z)\, z^2 + (w_t^T \cdot w_t^T) + 2\,(w_t^z \cdot w_t^T)\, z,
\end{aligned}$$

where $\mathbb{W}^{x,y}_{ijh} \equiv W^x_{ij} W^y_{ih}$.

Proof. The first identity follows from Lemma 1(i) and (iii), using the symmetry of $\mathbb{T}$ to combine the cross terms. For the second, expand the componentwise square of w_{1t} and use $\sum_{j,h} W^x_{ij} W^y_{ih}\, x_j y_h = (\mathbb{W}^{x,y} \times_3 y \times_2 x)_i$. □

The third lemma expands the drift term of Itô's formula, $\sum_j \partial_{S_j} w\, \mu_j^S$, when both w and the state drift are expanded to second order.

Lemma 3. *Consider a state $S = S^{ss} + \epsilon s$, with $s \in \{n, k, \chi\}$ and drift $\mu^S = \epsilon\, \mu^s_{1t} + \epsilon^2 \mu^s_{2t}$. Its first-order drift is $\mu^s_{1t} = \sum_x \Theta^{s,x}\, x + \Theta^{s,z}_t\, z + \Theta^{s,T}_t$, with matrices $\Theta^{s,x}$ and vectors $\Theta^{s,z}_t$, $\Theta^{s,T}_t$. Consider also a*

function $w = w^{ss} + \epsilon\, w_1 + \frac{\epsilon^2}{2} w_2$ *of the form* (16)*, with first-order matrices* W^x*, second-order tensors* $\mathcal{W}^{x,y}$*, and cross matrices* $W_t^{x,z}$*,* $W_t^{x,T}$*. Then, to second order:*

$$\sum_j \partial_{S_j} w\, \mu_j^S = \epsilon\, W^s \mu_{1t}^s + \frac{\epsilon^2}{2}\big[2\, W^s \mu_{2t}^s + (\partial_s w_2)\, \mu_{1t}^s\big],$$

where $\partial_s w_2 \equiv [\partial w_{2,i}/\partial s_j]_{ij}$ *is the Jacobian of* w_2 *with respect to* s*, and*

$$\begin{aligned}
(\partial_s w_2)\, \mu_{1t}^s \;=\; & \sum_{x,y} \big[\mathcal{W}^{s,y} \times_2 \Theta^{s,x} + \mathcal{W}^{x,s} \times_3 \Theta^{s,y}\big] \times_3 y \times_2 x \\
& + 2 \sum_x \Big[\tfrac{1}{2}\big(\mathcal{W}^{s,x} \times_2 \Theta_t^{s,z} + \mathcal{W}^{x,s} \times_3 \Theta_t^{s,z}\big) + W_t^{s,z} \Theta^{s,x}\Big] x\, z \\
& + 2 \sum_x \Big[\tfrac{1}{2}\big(\mathcal{W}^{s,x} \times_2 \Theta_t^{s,T} + \mathcal{W}^{x,s} \times_3 \Theta_t^{s,T}\big) + W_t^{s,T} \Theta^{s,x}\Big] x \\
& + 2\, W_t^{s,z} \Theta_t^{s,z}\, z^2 + 2\, W_t^{s,T} \Theta_t^{s,T} + 2\big[W_t^{s,z} \Theta_t^{s,T} + W_t^{s,T} \Theta_t^{s,z}\big] z.
\end{aligned}$$

Proof. Since $S = S^{ss} + \epsilon s$, we have $\partial_{S_j} = \epsilon^{-1} \partial_{s_j}$ and $\partial_{s_j} w_{1,i} = W_{ij}^s$, so $\partial_S w = W^s + \frac{\epsilon}{2} \partial_s w_2$. Multiplying by $\mu^S = \epsilon \mu_{1t}^s + \epsilon^2 \mu_{2t}^s$ gives the first expression, with a remainder of order ϵ^3. For the second expression, differentiate (16) with respect to s_j. In the double sum, the pairs with $x = s$ and those with $y = s$ both contribute, so $\partial w_2 / \partial s_j = \sum_y (\mathcal{W}^{s,y} \times_3 y)_{\cdot j} + \sum_x (\mathcal{W}^{x,s} \times_2 x)_{\cdot j} + 2 W_{\cdot j,t}^{s,z} z + 2 W_{\cdot j,t}^{s,T}$. Multiplying by $\mu_{1t,j}^s$, summing over j and applying Lemma 1(iii) gives the result. □

The last lemma expands a function that is log-linear in the states to second order.

Lemma 4. *Let* $f_i = f_i^{ss}\, e^{L_i}$ *with constant log-state loadings,* $L_i = \mathrm{e}_\chi \chi_i + \mathrm{e}_k (\mathcal{K}_i - \mathcal{K}_i^{ss}) + \mathrm{e}_{\log N} (\log N_i - \log N_i^{ss})$*, and define* $\mathrm{e}_n \equiv \mathrm{e}_{\log N} / N_i^{ss}$*. Its second-order expansion is:*

$$f_t = f^{ss} + \epsilon \sum_x D_{fx}\, x + \frac{\epsilon^2}{2} \sum_{x,y} \mathbb{D}_{fxy} \times_3 y \times_2 x,$$

where $D_{fx} \equiv \boldsymbol{diag}(f_i^{ss}\, \mathrm{e}_x)$*. A transformation* $u(f_i)$ *admits the same expansion, with first-order coefficients* $D_{u(f)x} = \boldsymbol{diag}\big(u'(f_i^{ss}) f_i^{ss}\, \mathrm{e}_x\big)$*. The second-order tensors are:*

$$\begin{aligned}
\mathbb{D}_{fxy} &= \boldsymbol{diag}_3\big(f_i^{ss}\, \mathrm{e}_x \mathrm{e}_y\big) - \mathbb{1}_{x=y=n}\, \boldsymbol{diag}_3\Big(\frac{f_i^{ss}\, \mathrm{e}_{\log N}}{(N_i^{ss})^2}\Big), \\
\mathbb{D}_{u(f)xy} &= \boldsymbol{diag}_3\Big(f_i^{ss}\big[u''(f_i^{ss}) f_i^{ss} + u'(f_i^{ss})\big]\mathrm{e}_x \mathrm{e}_y\Big) - \mathbb{1}_{x=y=n}\, \boldsymbol{diag}_3\Big(\frac{u'(f_i^{ss}) f_i^{ss}\, \mathrm{e}_{\log N}}{(N_i^{ss})^2}\Big).
\end{aligned}$$

Proof. In terms of state deviations, $L_i = \epsilon(\mathrm{e}_\chi \widehat{\chi}_i + \mathrm{e}_k k_i) + \mathrm{e}_{\log N} \log(1 + \epsilon n_i / N_i^{ss})$. Expanding the logarithm and the exponential to second order gives the first tensor. The correction for the (n, n)

pair arises because $\partial \log N / \partial N = 1/N$ varies with population, unlike the other loadings. Applying the chain rule twice to $u(f_i)$ gives the second tensor. □

C.2 Master Equations

We express the Master Equations of Proposition 3 in terms of the aggregate states. For the diversified economy, we derive the diffusion loadings of capital prices and aggregate consumption, and use them to characterize the interest rate adjustment $\mathcal{R}_{it}$ and covariance term ι^Q_{it}. We then derive the corresponding Master Equation for the island economy.

We consider the aggregate state variables $(z_t, \chi_t, N_t, \mathcal{K}_t, t)$, where calendar time captures the warming path T_t. We make explicit the dependence of workers' values and the value of local capital on these aggregate variables, $V_{it} = V_i(z_t, \chi_t, N_t, \mathcal{K}_t, t)$ and $Q_{it} = Q_i(z_t, \chi_t, N_t, \mathcal{K}_t, t)$.

Combining the aggregate shock of Section 2, the productivity process (1), the population law (8), and capital accumulation (4) with the investment rule of Proposition 1, the state evolves according to:

$$dz_t = -\theta z_t\, dt + dW_t, \qquad d\chi_{jt} = \left(X_{jt} - \kappa \chi_{jt}\right) dt + \Sigma_{\chi,jt}\, dW_{\chi,jt},$$
$$dN_{jt} = \left(M^*(V_t) N_t\right)_j dt, \qquad d\mathcal{K}_{jt} = \left(c_{j0} Q_{jt}^{\zeta} - \Delta_{jt} - \frac{\Sigma^2_{\Delta,jt}}{2}\right) dt - \Sigma_{\Delta,jt}\, dW_{\Delta,jt}.$$

For any smooth function F of the state, the generator $\mathcal{L}_t$ in (15) satisfies $\mathbb{E}_t[dF_t]/dt = \mathcal{L}_t F$. We decompose it as $\mathcal{L}_t = \mathcal{L}_{d,t} + \mathcal{L}_{o,t}$, separating the drift and Itô terms. The former contains the time derivative and first-order state derivatives, including $-\theta z\, \partial_z$, while the latter collects second-order derivatives, including $\frac{1}{2}\partial_{zz}$. Since local covariance matrices are of order ϵ^2 and the first-order expansions of V and Q are linear in z, the Itô terms contribute only at second order. Thus, $\mathcal{L}_{o,t}$ enters the SAME but not the FAME.

Using $\mathbb{E}_t[dV_{it}]/dt = \mathcal{L}_t V_i$ in the worker Bellman equation (6) gives the worker Master Equation in Proposition 3. Capitalists enter this equation through the investment rate in the capital drift of $\mathcal{L}_t$.

Capital prices depend on the aggregate state and evolve according to:

$$dQ_{it} = \mu^Q_{it}\, dt + \sigma^Q_{z,i,t}\, dW_t + \sum_\ell \sigma^Q_{\chi,i,\ell,t}\, dW_{\chi,\ell t} + \sum_\ell \sigma^Q_{\Delta,i,\ell,t}\, dW_{\Delta,\ell t}, \tag{C1}$$

where Itô's lemma gives the drift and diffusion loadings:

$$\mu_{it}^{Q} = \mathcal{L}_t Q_i, \qquad \sigma_{z,i,t}^{Q} = \frac{\partial Q_i}{\partial z}, \qquad \sigma_{\chi,i,\ell,t}^{Q} = \Sigma_{\chi,\ell t}\,\frac{\partial Q_i}{\partial \chi_\ell}, \qquad \sigma_{\Delta,i,\ell,t}^{Q} = -\Sigma_{\Delta,\ell t}\,\frac{\partial Q_i}{\partial \mathcal{K}_\ell}. \tag{C2}$$

These expressions connect the first derivatives obtained from the FAME to the risk terms in the SAME, which are quadratic in the loadings.

Aggregate capitalist consumption in Proposition 1 is:

$$\mathcal{C}(\chi, N, \mathcal{K}, Q) \equiv \sum_i \mathcal{C}_i\big(\chi_i, e^{\mathcal{K}_i}, N_i, Q_i\big) = \sum_i \Big[R_{0i}\, e^{\chi_i + (1-\phi)\mathcal{K}_i} N_i^{\psi} - \frac{\zeta c_{i0}}{1+\zeta}\, e^{\mathcal{K}_i} Q_i^{1+\zeta}\Big]. \tag{C3}$$

Accounting for the state dependence of Q, the total derivative with respect to x_ℓ, for $x \in \{\chi, N, \mathcal{K}\}$, is $\mathcal{C}_{\ell,x} + \sum_i \mathcal{C}_{i,Q}\,\partial Q_i/\partial x_\ell$. Derivatives with respect to log capital satisfy $\mathcal{C}_{i,\mathcal{K}} = K_i \mathcal{C}_{i,K}$, $\mathcal{C}_{i,\mathcal{K}\mathcal{K}} = K_i^2 \mathcal{C}_{i,KK} + K_i \mathcal{C}_{i,K}$, and $\mathcal{C}_{i,\mathcal{K}Q} = K_i \mathcal{C}_{i,KQ}$.

Applying Itô's lemma gives the diffusion loadings of consumption growth:

$$\begin{aligned} \sigma_{z,t}^{C} &= \frac{1}{C_t}\sum_i \mathcal{C}_{i,Q}\,\sigma_{z,i,t}^{Q}, \qquad \sigma_{\chi,\ell,t}^{C} = \frac{1}{C_t}\Big[\mathcal{C}_{\ell,\chi}\,\Sigma_{\chi,\ell t} + \sum_i \mathcal{C}_{i,Q}\,\sigma_{\chi,i,\ell,t}^{Q}\Big], \\ \sigma_{\Delta,\ell,t}^{C} &= \frac{1}{C_t}\Big[\sum_i \mathcal{C}_{i,Q}\,\sigma_{\Delta,i,\ell,t}^{Q} - \mathcal{C}_{\ell,K}\,K_{\ell t}\Sigma_{\Delta,\ell t}\Big]. \end{aligned} \tag{C4}$$

The derivatives entering these loadings and the consumption drift are:

$$\begin{aligned} \mathcal{C}_{i,\chi} &= R_{it}K_{it}, & \mathcal{C}_{i,K} &= (1-\phi)\,R_{it} - \frac{\zeta c_{i0}}{1+\zeta}\,Q_{it}^{1+\zeta}, \\ \mathcal{C}_{i,N} &= \frac{\psi\,R_{it}K_{it}}{N_{it}}, & \mathcal{C}_{i,Q} &= -\zeta c_{i0}\,K_{it}Q_{it}^{\zeta}, \\ \mathcal{C}_{i,\chi\chi} &= R_{it}K_{it}, & \mathcal{C}_{i,KK} &= -\frac{\phi(1-\phi)\,R_{it}}{K_{it}}, \\ \mathcal{C}_{i,QQ} &= -\zeta^2 c_{i0}\,K_{it}Q_{it}^{\zeta-1}, & \mathcal{C}_{i,KQ} &= -\zeta c_{i0}\,Q_{it}^{\zeta}. \end{aligned}$$

Two cross terms vanish: $\mathcal{C}_{i,\chi Q} = 0$, while $\mathrm{Cov}_t(d\chi_{it}, dK_{it}) = 0$ because productivity and depreciation innovations are independent. At steady state, after converting capital derivatives to log units, these expressions give the consumption rows $\mathcal{C}_n, \mathcal{C}_k, \mathcal{C}_\chi, \mathcal{C}_Q$ in Appendix C.3 and the Hessian rows in Appendix C.4.

Using the loadings in (C4), the variance rate of consumption growth is:

$$\|\sigma_t^C\|^2 \equiv (\sigma_{z,t}^C)^2 + \sum_{\ell,m} \mathfrak{c}_{\chi,\ell m}\,\sigma_{\chi,\ell,t}^C \sigma_{\chi,m,t}^C + \sum_{\ell,m} \mathfrak{c}_{\Delta,\ell m}\,\sigma_{\Delta,\ell,t}^C \sigma_{\Delta,m,t}^C.$$

We define the variance rate $\|\sigma^Q_{i,t}\|^2$ analogously, using the capital-price loadings in (C2).

Let μ^x_i denote the drift of state x_i, for $x \in \{\chi, N, \mathcal{K}\}$. The consumption-growth drift satisfies $\mu^C_t \mathcal{C} = \sum_x \sum_i \mathcal{C}_{i,x} \mu^x_i + \sum_i \mathcal{C}_{i,Q} \mathcal{L}_t Q_i + \iota^C_t$, where the Itô correction is:

$$\iota^C_t \equiv \frac{1}{2} \sum_{x \in \{\chi, \mathcal{K}\}} \sum_i \mathcal{C}_{i,xx} (\sigma^x_{i,t})^2 + \frac{1}{2} \sum_i \mathcal{C}_{i,QQ} \|\sigma^Q_{i,t}\|^2 - \sum_i \mathcal{C}_{i,\mathcal{K}Q} \Sigma_{\Delta,it} \sum_\ell \mathfrak{c}_{\Delta,i\ell} \sigma^Q_{\Delta,i,\ell,t}. \tag{C5}$$

Here, $(\sigma^\chi_{i,t})^2 = \Sigma^2_{\chi,it}$ and $(\sigma^{\mathcal{K}}_{i,t})^2 = \Sigma^2_{\Delta,it}$. Substituting the consumption drift and variance into the risk-free rate in Proposition 1 gives:

$$\overline{R}_t = \rho + \frac{\overline{\gamma}}{\mathcal{C}} \Big[\sum_{x \in \{\chi, N, \mathcal{K}\}} \sum_i \mathcal{C}_{i,x} \mu^x_i + \sum_i \mathcal{C}_{i,Q} \mathcal{L}_t Q_i \Big] + \frac{\overline{\gamma} \iota^C_t}{\mathcal{C}} - \frac{\overline{\gamma}(\overline{\gamma}+1)}{2} \|\sigma^C_t\|^2. \tag{C6}$$

Using $\mathbb{E}_t[dQ_{it}]/dt = \mathcal{L}_t Q_i$, we write the pricing equation in Proposition 1 as:

$$(\overline{R}_t + \Delta_{it}) Q_i = R_i + \frac{c_{i0}}{1+\zeta} Q_i^{1+\zeta} + \mathcal{L}_t Q_i + \Upsilon^Q_i,$$

where Υ^Q_i collects the covariance terms in (B19). We evaluate them using the stochastic discount factor (B21) and the state-space loadings (C2) and (C4). The covariance between the kernel and capital-price growth is:

$$\begin{aligned} \frac{1}{dt} \mathbb{C}\mathrm{ov}_t \left[\frac{d\Lambda_t}{\Lambda_t}, \frac{dQ_{it}}{Q_{it}} \right] \;=\; & -\frac{\overline{\gamma}}{Q_{it}} \Big[\sigma^C_{z,t} \sigma^Q_{z,i,t} + \sum_{\ell,m} \mathfrak{c}_{\chi,\ell m} \sigma^C_{\chi,\ell,t} \sigma^Q_{\chi,i,m,t} \\ & + \sum_{\ell,m} \mathfrak{c}_{\Delta,\ell m} \sigma^C_{\Delta,\ell,t} \sigma^Q_{\Delta,i,m,t} \Big]. \end{aligned} \tag{C7}$$

The second covariance captures the direct exposure of capital to depreciation risk. Using the diffusion $-\Sigma_{\Delta,it}\, dW_{\Delta,it}$ of dK_{it}/K_{it} in (B9), we obtain:

$$\frac{1}{dt} \mathbb{C}\mathrm{ov}_t \left[\frac{d\Lambda_t}{\Lambda_t} + \frac{dQ_{it}}{Q_{it}}, \frac{dK_{it}}{K_{it}} \right] = \overline{\gamma} \Sigma_{\Delta,it} \sum_\ell \mathfrak{c}_{\Delta,i\ell} \sigma^C_{\Delta,\ell,t} - \frac{\Sigma_{\Delta,it}}{Q_{it}} \sum_\ell \mathfrak{c}_{\Delta,i\ell} \sigma^Q_{\Delta,i,\ell,t}. \tag{C8}$$

Adding these covariances and multiplying by Q_{it} gives:

$$\begin{aligned} \Upsilon^Q_i \;=\; & -\overline{\gamma} \sigma^C_{z,t} \sigma^Q_{z,i,t} - \overline{\gamma} \sum_{\ell,m} \mathfrak{c}_{\chi,\ell m} \sigma^C_{\chi,\ell,t} \sigma^Q_{\chi,i,m,t} - \overline{\gamma} \sum_{\ell,m} \mathfrak{c}_{\Delta,\ell m} \sigma^C_{\Delta,\ell,t} \sigma^Q_{\Delta,i,m,t} \\ & - \Sigma_{\Delta,it} \sum_\ell \mathfrak{c}_{\Delta,i\ell} \sigma^Q_{\Delta,i,\ell,t} + \overline{\gamma}\, Q_{it} \Sigma_{\Delta,it} \sum_\ell \mathfrak{c}_{\Delta,i\ell} \sigma^C_{\Delta,\ell,t}. \end{aligned} \tag{C9}$$

The first line captures the price channel, $-\overline{\gamma}\, \mathbb{C}\mathrm{ov}_t(dQ_{it}, dC_t/C_t)/dt$. The second contains the

hedge, $\mathbb{C}\text{ov}_t(dQ_{it}, dK_{it}/K_{it})/dt$, and the local risk term, $-\overline{\gamma}Q_{it}\,\mathbb{C}\text{ov}_t(dK_{it}/K_{it}, dC_t/C_t)/dt$. The double sums account for spatial correlations and reduce to single sums when shocks are independent across locations.

Substituting the risk-free rate (C6) into the pricing equation and regrouping gives the capital Master Equation in Proposition 3:

$$\rho Q_i = W_{it} + \mathcal{L}_t Q_i + \mathcal{R}_{it} + \iota^Q_{it},$$

where:

$$W_{it} \equiv R_i\big(\chi_i, e^{\mathcal{K}_i}, N_i\big) + \frac{c_{i0}}{1+\zeta} Q_i^{1+\zeta} - \Delta_{it} Q_i, \tag{C10}$$

$$\mathcal{R}_{it} \equiv -\,Q_i \,\frac{\overline{\gamma}}{\mathcal{C}}\Big[\sum_{x\in\{\chi,N,\mathcal{K}\}}\sum_j \mathcal{C}_{j,x}\,\mu^x_j + \sum_j \mathcal{C}_{j,Q}\,\mathcal{L}_t Q_j\Big], \tag{C11}$$

$$\iota^Q_{it} \equiv \Upsilon^Q_i - \frac{\overline{\gamma}\,\iota^C_t}{\mathcal{C}}\,Q_i + \frac{\overline{\gamma}(\overline{\gamma}+1)}{2}\,\|\sigma^C_t\|^2\,Q_i. \tag{C12}$$

Here, W_{it} is the flow payoff net of depreciation. The adjustment $\mathcal{R}_{it}$ captures how state and capital-price drifts affect valuation through the risk-free rate, linking locations through aggregate consumption. The term ι^Q_{it} combines the covariance correction Υ^Q_i with the explicit Itô and variance terms in (C6).

Island model. The main departure in the island model is that the generator $\mathcal{L}_t$ uses the investment policy $g_j(Q_j, R_j)$ in the drift of log capital. The loadings in (C2) now describe the island valuation Q. To write the value equation of Proposition 2 as a Master Equation, define:

$$\Omega_i(Q,R) \equiv \max_g \left\{\frac{Q^{\overline{\gamma}}\big[R - c_i(g)\big]^{1-\overline{\gamma}}}{1-\overline{\gamma}} + gQ\right\} + \frac{\overline{\gamma}\rho}{\overline{\gamma}-1}Q, \tag{C13}$$

where $g_i(Q,R)$ is the optimal investment rate. The maximization corresponds to the Bellman flow in (B26) multiplied by $Q^{\overline{\gamma}}$, while the final term adjusts the discount rate from $\rho/(1-\overline{\gamma})$ to ρ. Multiplying (B28) by Q_{it} and adding this adjustment to both sides yields:

$$\begin{aligned}\rho Q_{it} \;=\; & \Omega_i\big(Q_{it}, R_i(\chi_i, e^{\mathcal{K}_i}, N_i)\big) - \Big[\Delta_{it} + \frac{\overline{\gamma}\,\Sigma^2_{\Delta,it}}{2}\Big]Q_{it} + (1-\overline{\gamma})\sum_j \Sigma_{\Delta,ijt}\,\frac{\partial Q_{it}}{\partial \mathcal{K}_j} \\ & - \frac{\overline{\gamma}}{2Q_{it}}\,\|\sigma^Q_{it}\|^2 + \mathcal{L}_t Q_{it}.\end{aligned} \tag{C14}$$

The variance rate of dQ_{it} is:

$$\|\sigma_{it}^{Q}\|^2 \equiv \left(\frac{\partial Q_{it}}{\partial z}\right)^2 + \sum_{j,\ell} \Sigma_{\chi,j\ell t} \frac{\partial Q_{it}}{\partial \chi_j} \frac{\partial Q_{it}}{\partial \chi_\ell} + \sum_{j,\ell} \Sigma_{\Delta,j\ell t} \frac{\partial Q_{it}}{\partial \mathcal{K}_j} \frac{\partial Q_{it}}{\partial \mathcal{K}_\ell}.$$

The variance term in (C14) arises from applying Itô's lemma to the nonlinear value function $(Q_{it}K)^{1-\overline{\gamma}}/(1-\overline{\gamma})$. Second derivatives of Q_{it} enter through $\mathcal{L}_t Q_{it}$, while products of its first derivatives form $\|\sigma_{it}^{Q}\|^2$.

Unlike the diversified economy, the island model has no risk-free-rate adjustment. Instead, the value equation directly captures local depreciation risk, the hedge, and fluctuations in the value of capital.

C.3 FAME: Proof of Proposition 4

We expand the Master Equations of Proposition 3 to first order and collect coefficients into the four FAME blocks and the linearized laws of motion. Following the conventions of Appendix C.1, we write the first-order terms in (16) and (17) as $v_{1t} = \sum_x V^x x + v_t^z z + v_t^T$ and $q_{1t} = \sum_x Q^x x + q_t^z z + q_t^T$, for $x \in \{\chi, k, n\}$.

We present the derivation for the diversified economy, where Q is the price of capital. We relegate the modifications for the island economy to the end. Appendix H.1 provides the corresponding changes under Epstein-Zin preferences.

C.3.1 First-Order Approximations

We linearize the flow payoffs, continuation values, and laws of motion for productivity, capital, and population. Throughout, we omit terms of order ϵ^2 and higher.

Workers' flow payoff. The first-order change in workers' flow payoffs is:

$$U_t - U^{ss} = \epsilon \sum_x \mathrm{D}_{Ux}\, x, \qquad \mathrm{D}_{Ux} \equiv \mathbf{diag}\big(u'(\widetilde{C}_i^{ss})\, \widetilde{C}_i^{ss}\, \mathrm{e}_{x,i}^{C}\big),$$

where $\mathrm{e}_{\chi,i}^{C} = \eta + 1 - \beta$, $\mathrm{e}_{k,i}^{C} = \xi(1-\omega-\varpi)$, and $\mathrm{e}_{n,i}^{C} = -\xi(1-\varpi)/N_i^{ss}$.

Capitalists' flow payoff. For capitalists, the corresponding expansion is:

$$W_t - W^{ss} = \epsilon \Big[\sum_x \mathrm{D}_{Qx}\, x + \mathrm{d}_{QT}\, \widehat{T}_t \Big],$$
$$\mathrm{D}_{Qx} = \mathrm{D}_{Rx} \equiv \mathbf{diag}\big(R_i^{ss}\, \mathrm{e}^R_{x,i}\big), \qquad \mathrm{d}_{QT} \equiv -\mathbf{vec}\big(Q_i^{ss}\, \delta_i\big), \tag{C15}$$

where $\mathrm{e}^R_{\chi,i} = 1$, $\mathrm{e}^R_{k,i} = -\phi$, $\mathrm{e}^R_{n,i} = \psi / N_i^{ss}$, and $\widehat{T}_t \equiv T_t + \sigma_t z$.

Continuation value from migration. Let $M \equiv M(V^{ss}) = \mu\big(m(V^{ss}) - \mathbf{I}\big)$ denote the Jacobian of the migration operator $\mathcal{M}[V]$ at steady state, the transpose of the matrix $M^*(V^{ss})$ of Section 2. The first-order change in the continuation value from migration is:

$$\mathcal{M}[V] - \mathcal{M}[V^{ss}] = \epsilon M v_{1t}.$$

Continuation value from aggregate shocks. Since z_t mean-reverts at rate θ and the first-order terms are linear in z, we obtain:

$$\mathcal{A}(z)[\epsilon v_t^z z] = -\epsilon\theta v_t^z z, \qquad \mathcal{A}(z)[\epsilon q_t^z z] = -\epsilon\theta q_t^z z.$$

Continuation value from changes in the productivity distribution. The covariance terms enter only at second order, so the first-order contributions to workers' and capitalists' continuation values are:

$$\sum_j (X_{jt} - \kappa\chi_j)\frac{\partial V}{\partial \chi_j} + \frac{1}{2}\sum_{j,\ell}\Sigma_{\chi,j\ell t}\frac{\partial^2 V}{\partial\chi_j\partial\chi_\ell} = \epsilon V^\chi\big(-\kappa\,\widehat{\chi} + \mathrm{d}_{XT}\,\widehat{T}_t\big),$$
$$\sum_j (X_{jt} - \kappa\chi_j)\frac{\partial Q}{\partial \chi_j} + \frac{1}{2}\sum_{j,\ell}\Sigma_{\chi,j\ell t}\frac{\partial^2 Q}{\partial\chi_j\partial\chi_\ell} = \epsilon Q^\chi\big(-\kappa\,\widehat{\chi} + \mathrm{d}_{XT}\,\widehat{T}_t\big),$$

where $\widehat{\chi} \equiv \chi/\epsilon$ is the scaled productivity deviation and

$$\mathrm{d}_{XT} \equiv \kappa\, \mathbf{vec}(\chi_i). \tag{C16}$$

Continuation value from changes in the capital distribution. Similarly, linearizing the capital drift and omitting second-order risk terms gives:

$$\begin{aligned}\sum_j \Big(c_{j0}Q_{jt}^{\zeta} - \Delta_{jt} - \frac{\Sigma_{\Delta,jt}^2}{2}\Big)\frac{\partial V}{\partial \mathcal{K}_j} + \frac{1}{2}\sum_{j,\ell}\Sigma_{\Delta,j\ell t}\frac{\partial^2 V}{\partial \mathcal{K}_j \partial \mathcal{K}_\ell} &= \epsilon V^k\big(\mathrm{D}_{\mathcal{I}Q}\, q_{1t} + \mathrm{d}_{\mathcal{I}T}\,\widehat{T}_t\big),\\ \sum_j \Big(c_{j0}Q_{jt}^{\zeta} - \Delta_{jt} - \frac{\Sigma_{\Delta,jt}^2}{2}\Big)\frac{\partial Q}{\partial \mathcal{K}_j} + \frac{1}{2}\sum_{j,\ell}\Sigma_{\Delta,j\ell t}\frac{\partial^2 Q}{\partial \mathcal{K}_j \partial \mathcal{K}_\ell} &= \epsilon Q^k\big(\mathrm{D}_{\mathcal{I}Q}\, q_{1t} + \mathrm{d}_{\mathcal{I}T}\,\widehat{T}_t\big),\end{aligned}$$

where

$$\mathrm{D}_{\mathcal{I}Q} \equiv \mathbf{diag}\big(\zeta c_{i0}(Q_i^{ss})^{\zeta-1}\big) = \mathbf{diag}\Big(\frac{\zeta\Delta_i}{Q_i^{ss}}\Big), \qquad \mathrm{d}_{\mathcal{I}T} \equiv -\mathbf{vec}(\delta_i). \tag{C17}$$

Continuation value from changes in the population distribution. Differentiating the migration shares in (7) gives:

$$\frac{\partial m_{ji}}{\partial V_k} = \nu\, m_{ji}\big(\mathbb{1}_{i=k} - m_{jk}\big).$$

Evaluating at steady state, we obtain $M^*(V^{ss} + dV)\,N^{ss} = M^* N^{ss} + G\,dV$ to first order, where

$$G \equiv \nu\mu\Big(\mathbf{diag}(m^* N^{ss}) - m^*\mathbf{diag}(N^{ss})\,m\Big). \tag{C18}$$

Therefore, the first-order continuation-value terms are:

$$\begin{aligned}\sum_j \big(M^*(V_t)N_t\big)_j\frac{\partial V}{\partial N_j} &= \epsilon V^n\big(M^*\,n + G\,v_{1t}\big),\\ \sum_j \big(M^*(V_t)N_t\big)_j\frac{\partial Q}{\partial N_j} &= \epsilon Q^n\big(M^*\,n + G\,v_{1t}\big).\end{aligned}$$

Risk-free rate (diversified model only). Since the drift terms in (C11) are first order, we evaluate $Q_i/\mathcal{C}$ and the consumption derivatives of (C3) at the steady state. Substituting the first-order drifts of the previous paragraphs and grouping terms, we get up to first order:

$$\mathcal{R}_{1t} = -\Big[\mathrm{D}_{rN}\,n + \mathrm{D}_{r\chi}\,\widehat{\chi} + \mathrm{d}_{rT}\,\widehat{T}_t + \mathrm{D}_{rV}\,v_{1t} + \mathrm{D}_{rQ}\,q_{1t} + \mathrm{D}_{r\mu}\,(\mathcal{L}_t Q)_1\Big], \tag{C19}$$

where $(\mathcal{L}_t Q)_1$ collects the first-order capital-price drifts, expanded later in the next section, and:

$$\begin{aligned}
\mathrm{D}_{rN} &\equiv \frac{\overline{\gamma}}{C^{ss}}\, Q^{ss}\mathcal{C}_n\, M^*, & \mathrm{D}_{r\chi} &\equiv -\frac{\overline{\gamma}\kappa}{C^{ss}}\, Q^{ss}\mathcal{C}_\chi, & \mathrm{d}_{rT} &\equiv \frac{\overline{\gamma}}{C^{ss}}\big(\mathcal{C}_k \mathrm{d}_{\mathcal{I}T} + \mathcal{C}_\chi \mathrm{d}_{XT}\big) Q^{ss},\\
\mathrm{D}_{rV} &\equiv \frac{\overline{\gamma}}{C^{ss}}\, Q^{ss}\mathcal{C}_n\, G, & \mathrm{D}_{rQ} &\equiv \frac{\overline{\gamma}}{C^{ss}}\, Q^{ss}\mathcal{C}_k\, \mathrm{D}_{\mathcal{I}Q}, & \mathrm{D}_{r\mu} &\equiv \frac{\overline{\gamma}}{C^{ss}}\, Q^{ss}\mathcal{C}_Q,\\
\mathcal{C}_n &\equiv \Big(\frac{\psi R_i^{ss} K_i^{ss}}{N_i^{ss}}\Big)_i^*, & \mathcal{C}_\chi &\equiv \big(R_i^{ss} K_i^{ss}\big)_i^*, & \mathcal{C}_Q &\equiv \big(-\zeta \Delta_i K_i^{ss}\big)_i^*,\\
\mathcal{C}_k &\equiv \Big((1-\phi) R_i^{ss} K_i^{ss} - \frac{\zeta}{1+\zeta}\Delta_i Q_i^{ss} K_i^{ss}\Big)_i^*. & & & & & \text{(C20)}
\end{aligned}$$

The rows $\mathcal{C}_n, \mathcal{C}_k, \mathcal{C}_\chi, \mathcal{C}_Q$ are the $1 \times \mathcal{I}$ derivatives of aggregate consumption at the steady state, $\mathcal{C}_k$ with respect to log capital, and $Q^{ss}\mathcal{C}_x$ is the outer product with entries $Q_i^{ss}(\mathcal{C}_x)_j$.

Itô correction (diversified model only). All terms in the correction ι_i^Q are second-order terms and therefore do not enter the FAME.

C.3.2 FAME in Vector Notation

We now collect the first-order approximations into the linearized laws of motion and the four blocks of Proposition 4.

We first define the coefficients of the state drifts, for each state $s \in \{n, k, \chi\}$ and deviation $x \in \{n, k, \chi\}$:

$$\begin{aligned}
\Theta^{n,x} &\equiv \mathbb{1}_{x=n}\, M^* + G\, V^x, & \Theta_t^{n,z} &\equiv G\, v_t^z, & \Theta_t^{n,T} &\equiv G\, v_t^T,\\
\Theta^{k,x} &\equiv \mathrm{D}_{\mathcal{I}Q}\, Q^x, & \Theta_t^{k,z} &\equiv \mathrm{D}_{\mathcal{I}Q}\, q_t^z + \mathrm{d}_{\mathcal{I}T}\, \sigma_t, & \Theta_t^{k,T} &\equiv \mathrm{D}_{\mathcal{I}Q}\, q_t^T + \mathrm{d}_{\mathcal{I}T}\, T_t,\\
\Theta^{\chi,x} &\equiv -\kappa\, \mathbb{1}_{x=\chi}\, \mathbf{I}, & \Theta_t^{\chi,z} &\equiv \mathrm{d}_{XT}\, \sigma_t, & \Theta_t^{\chi,T} &\equiv \mathrm{d}_{XT}\, T_t.
\end{aligned}$$

The blocks $\Theta^{s,x}$ form the $3\mathcal{I} \times 3\mathcal{I}$ transition matrix $\Theta \equiv [\Theta^{s,x}]_{s,x}$, while the remaining coefficients are stacked in Θ_t^z and Θ_t^T.

Substituting the first-order expansions into the worker Master Equation and identifying the coefficients of x, z, and the constant components gives:

$$\begin{aligned}
\rho\, V^x &= \mathrm{D}_{Ux} + M V^x + \sum_s V^s\, \Theta^{s,x},\\
(\rho + \theta)\, v_t^z - \partial_t v_t^z &= M v_t^z + \sum_s V^s\, \Theta_t^{s,z},\\
\rho\, v_t^T - \partial_t v_t^T &= M v_t^T + \sum_s V^s\, \Theta_t^{s,T},
\end{aligned}$$

for each $x \in \{n, k, \chi\}$. The sums over s capture the continuation values associated with changes in population, capital, and productivity.

For capitalists, we must also account for the risk-free-rate adjustment (C19). In particular, the first-order drift of capital prices is:

$$\begin{aligned}(\mathcal{L}_t Q)_1 &= \sum_s Q^s \Big[\sum_x \Theta^{s,x} x + \Theta_t^{s,z} z + \Theta_t^{s,T} \Big] \\ &\quad + (\partial_t q_t^z - \theta q_t^z)\, z + \partial_t q_t^T.\end{aligned}$$

This drift enters the capital Master Equation directly and through the interest rate adjustment $-\mathrm{D}_{r\mu}(\mathcal{L}_t Q)_1$. Thus, the combined effect is weighted by $\mathbf{I} - \mathrm{D}_{r\mu}$. Using the flow (C15) and defining $\mathrm{D}_{rn} \equiv \mathrm{D}_{rN}$ and $\mathrm{D}_{rk} \equiv 0$, we obtain:

$$\begin{aligned}\rho\, Q^x &= \mathrm{D}_{Qx} - \mathrm{D}_{rx} - \mathrm{D}_{rV} V^x - \mathrm{D}_{rQ} Q^x + (\mathbf{I} - \mathrm{D}_{r\mu}) \sum_s Q^s\, \Theta^{s,x}, \\ \big(\rho\, \mathbf{I} + \theta\, (\mathbf{I} - \mathrm{D}_{r\mu})\big)\, q_t^z - (\mathbf{I} - \mathrm{D}_{r\mu})\, \partial_t q_t^z &= \mathrm{d}_{QT}\sigma_t - \mathrm{d}_{rT}\sigma_t - \mathrm{D}_{rV} v_t^z - \mathrm{D}_{rQ} q_t^z + (\mathbf{I} - \mathrm{D}_{r\mu}) \sum_s Q^s\, \Theta_t^{s,z}, \\ \rho\, q_t^T - (\mathbf{I} - \mathrm{D}_{r\mu})\, \partial_t q_t^T &= \mathrm{d}_{QT} T_t - \mathrm{d}_{rT} T_t - \mathrm{D}_{rV} v_t^T - \mathrm{D}_{rQ} q_t^T + (\mathbf{I} - \mathrm{D}_{r\mu}) \sum_s Q^s\, \Theta_t^{s,T}.\end{aligned}$$

The system is block recursive. We first solve jointly for the n and k coefficients, whose equations are quadratic. Given these solutions, the χ block is linear, followed by the stochastic and trend blocks, which are also linear in their respective time-dependent coefficients.

Deterministic FAME. We stack the worker and capitalist responses to population and capital in $\mathbf{V} = \begin{pmatrix} V^n & V^k \\ Q^n & Q^k \end{pmatrix}$ and define the matrix:

$$\widetilde{\mathbf{I}} \equiv \begin{pmatrix} \mathbf{I} & 0 \\ 0 & \mathbf{I} - \mathrm{D}_{r\mu} \end{pmatrix}. \tag{C21}$$

The deterministic FAME then takes the form:

$$\rho\,\mathbf{V} = \mathbf{D} + \mathbf{M}\,\mathbf{V} + \widetilde{\mathbf{I}}\big[\mathbf{V}\widetilde{\mathbf{M}} + \mathbf{V}\mathbf{G}\mathbf{V}\big], \tag{C22}$$

$$\mathbf{D} \equiv \begin{pmatrix} \mathrm{D}_{UN} & \mathrm{D}_{U\mathcal{K}} \\ \mathrm{D}_{QN} - \mathrm{D}_{rN} & \mathrm{D}_{Q\mathcal{K}} \end{pmatrix}, \qquad \mathbf{M} \equiv \begin{pmatrix} M & 0 \\ 0 & 0 \end{pmatrix} + \mathbf{D}_r, \qquad \mathbf{D}_r \equiv \begin{pmatrix} 0 & 0 \\ -\mathrm{D}_{rV} & -\mathrm{D}_{rQ} \end{pmatrix},$$

$$\widetilde{\mathbf{M}} \equiv \begin{pmatrix} M^* & 0 \\ 0 & 0 \end{pmatrix} + \mathbf{D}_{\mathcal{I}}, \qquad \mathbf{D}_{\mathcal{I}} \equiv 0, \qquad \mathbf{G} \equiv \begin{pmatrix} G & 0 \\ 0 & \mathrm{D}_{\mathcal{I}Q} \end{pmatrix}.$$

The matrix $\mathbf{D}_{\mathcal{I}}$ captures the direct response of investment to changes in the state. It vanishes in the diversified economy but is nonzero in the island economy, as shown below.

The matrix $\widetilde{\mathbf{I}}$ is invertible, since $\det(\mathbf{I} - \mathrm{D}_{r\mu}) = 1 - \frac{\overline{\gamma}}{C^{ss}}\mathcal{C}_Q Q^{ss} = 1 + \frac{\overline{\gamma}\zeta}{C^{ss}}\sum_i \Delta_i Q_i^{ss} K_i^{ss} \geq 1$. Since $\mathrm{D}_{r\mu}$ is a rank-one matrix, the Sherman-Morrison formula gives the inverse of $\widetilde{\mathbf{I}}$ explicitly:

$$\widetilde{\mathbf{I}}^{-1} = \begin{pmatrix} \mathbf{I} & 0 \\ 0 & (\mathbf{I} - \mathrm{D}_{r\mu})^{-1} \end{pmatrix}, \qquad (\mathbf{I} - \mathrm{D}_{r\mu})^{-1} = \mathbf{I} + \frac{\mathrm{D}_{r\mu}}{1 - \frac{\overline{\gamma}}{C^{ss}}\mathcal{C}_Q Q^{ss}}. \tag{C23}$$

Productivity FAME. Given $\mathbf{V}$, we construct the continuation matrix:

$$\overline{\Psi} \equiv \mathbf{M} + \widetilde{\mathbf{I}}\,\mathbf{V}\mathbf{G} = \begin{pmatrix} M + V^n G & V^k \mathrm{D}_{\mathcal{I}Q} \\ Q^n G - \big(\mathrm{D}_{rV} + \mathrm{D}_{r\mu} Q^n G\big) & Q^k \mathrm{D}_{\mathcal{I}Q} - \big(\mathrm{D}_{rQ} + \mathrm{D}_{r\mu} Q^k \mathrm{D}_{\mathcal{I}Q}\big) \end{pmatrix}. \tag{C24}$$

Stacking the productivity coefficients in $\mathbf{V}^\chi = (V^\chi; Q^\chi)$ and moving the mean-reversion terms to the left, we obtain:

$$\big(\rho\,\mathbf{I} + \kappa\,\widetilde{\mathbf{I}}\big)\,\mathbf{V}^\chi = \mathbf{D}^\chi + \overline{\Psi}\,\mathbf{V}^\chi, \qquad \mathbf{D}^\chi \equiv \begin{pmatrix} \mathrm{D}_{U\chi} \\ \mathrm{D}_{Q\chi} - \mathrm{D}_{r\chi} \end{pmatrix}.$$

Stochastic and trend FAME. The remaining two blocks share the continuation matrix $\overline{\Psi}$ and the forcing vector:

$$\Psi \equiv \mathbf{d}_T + \widetilde{\mathbf{I}}\big[\mathbf{V}^\chi\,\mathrm{d}_{XT} + \mathbf{V}^k\,\mathrm{d}_{\mathcal{I}T}\big], \qquad \mathbf{d}_T \equiv \begin{pmatrix} 0 \\ \mathrm{d}_{QT} - \mathrm{d}_{rT} \end{pmatrix}, \quad \mathbf{V}^k \equiv \begin{pmatrix} V^k \\ Q^k \end{pmatrix}. \tag{C25}$$

Defining $\mathbf{v}_t^z = (v_t^z; q_t^z)$ and $\mathbf{v}_t^T = (v_t^T; q_t^T)$, the stochastic and trend FAME satisfy:

$$\left(\rho\,\mathbf{I} + \theta\,\widetilde{\mathbf{I}}\right)\mathbf{v}_t^z - \widetilde{\mathbf{I}}\,\frac{\partial \mathbf{v}_t^z}{\partial t} = \Psi\,\sigma_t + \overline{\Psi}\,\mathbf{v}_t^z, \tag{C26}$$

$$\rho\,\mathbf{v}_t^T - \widetilde{\mathbf{I}}\,\frac{\partial \mathbf{v}_t^T}{\partial t} = \Psi\,T_t + \overline{\Psi}\,\mathbf{v}_t^T. \tag{C27}$$

First-order laws of motion. The drift coefficients above give the linearized laws of motion for the state deviations $\mathbf{s}_t = (n_t, k_t, \widehat{\chi}_t)$:

$$\dot{\mathbf{s}}_t = \Theta\,\mathbf{s}_t + \Theta_t^z\,z_t + \Theta_t^T, \tag{C28}$$

as stated in Proposition 4.

Island model. The derivations for the island economy differ from the ones of the diversified economy in three ways. First, there is no risk-free-rate adjustment, so we set $\mathrm{D}_{rN} = \mathrm{D}_{r\chi} = \mathrm{D}_{rV} = \mathrm{D}_{rQ} = \mathrm{D}_{r\mu} = 0$, $\mathrm{d}_{rT} = 0$, and $\widetilde{\mathbf{I}} = \mathbf{I}$ in the FAME equations above. Second, the capitalists' flow payoff changes. Third, so does the continuation value from changes in the capital distribution.

The flow in (C14) is $\Omega_i(Q,R) - [\Delta_{it} + \overline{\gamma}\Sigma_{\Delta,it}^2/2]Q$, with Ω_i defined in (C13). By the envelope theorem at $g = g_i(Q,R)$:

$$\begin{aligned}\Omega_{i,Q}(Q,R) &= \frac{\overline{\gamma}}{1-\overline{\gamma}}\,Q^{\overline{\gamma}-1}\left[R - c_i(g)\right]^{1-\overline{\gamma}} + g + \frac{\overline{\gamma}\rho}{\overline{\gamma}-1},\\ \Omega_{i,R}(Q,R) &= \left(\frac{Q}{R - c_i(g)}\right)^{\overline{\gamma}}.\end{aligned} \tag{C29}$$

At the steady state, $g_i = \Delta_i$, $R_i^{ss} - c_i(\Delta_i) = \rho c_i'(\Delta_i)$ and $Q_i^{ss} = \rho^{\overline{\gamma}/(\overline{\gamma}-1)}c_i'(\Delta_i)$, so:

$$\Omega_{i,Q}^{ss} = \Delta_i, \qquad \Omega_{i,R}^{ss} = \left(Q_i^{ss}\right)^{\overline{\gamma}}\left[\rho c_i'(\Delta_i)\right]^{-\overline{\gamma}} = \frac{Q_i^{ss}}{c_i'(\Delta_i)}. \tag{C30}$$

The Q derivative cancels the depreciation term $-\Delta_i Q$, so $\mathrm{D}_{QQ} = 0$ and the expansion has the form (C15) with $\mathrm{D}_{Qx} = \mathrm{D}_{QR}\mathrm{D}_{Rx}$, $\mathrm{D}_{QR} \equiv \mathbf{diag}\left(Q_i^{ss}/c_i'(\Delta_i)\right)$ and $\mathrm{d}_{QT} \equiv -\mathbf{vec}(Q_i^{ss}\delta_i)$.

Now, we focus on the continuation value from changes in the capital distribution. The invest-

ment rate $g_i(Q,R)$ is defined by the first-order condition of Proposition 2. Differentiating it:

$$\Big(c_i''(g)+\frac{\overline{\gamma}\,c_i'(g)^2}{R-c_i(g)}\Big)dg=\frac{(1-\overline{\gamma})\,c_i'(g)}{Q}\,dQ+\frac{\overline{\gamma}\,c_i'(g)}{R-c_i(g)}\,dR, \tag{C31}$$

and using $c_i''(\Delta_i)=c_i'(\Delta_i)/(\zeta\Delta_i)$ at the steady state:

$$g_{i,Q}^{ss}\equiv\frac{\partial g_i}{\partial Q}\Big|_{ss}=\frac{(1-\overline{\gamma})\,\rho\zeta\Delta_i}{Q_i^{ss}(\rho+\overline{\gamma}\zeta\Delta_i)},\qquad g_{i,R}^{ss}\equiv\frac{\partial g_i}{\partial R}\Big|_{ss}=\frac{\overline{\gamma}\zeta\Delta_i}{c_i'(\Delta_i)(\rho+\overline{\gamma}\zeta\Delta_i)}. \tag{C32}$$

With $g_j(Q_{jt},R_{jt})$ in place of $c_{j0}Q_{jt}^{\zeta}$ in the capital drift, linearizing and omitting second-order risk terms gives:

$$\begin{aligned}\sum_j\Big(g_j(Q_{jt},R_{jt})-\Delta_{jt}-\frac{\Sigma_{\Delta,jt}^2}{2}\Big)\frac{\partial V}{\partial\mathcal{K}_j}+\frac{1}{2}\sum_{j,\ell}\Sigma_{\Delta,j\ell t}\frac{\partial^2V}{\partial\mathcal{K}_j\partial\mathcal{K}_\ell}&=\epsilon V^k\Big(\mathrm{D}_{\mathcal{I}Q}\,q_{1t}+\sum_x\mathrm{D}_{\mathcal{I}x}\,x+\mathrm{d}_{\mathcal{I}T}\,\widehat{T}_t\Big),\\ \sum_j\Big(g_j(Q_{jt},R_{jt})-\Delta_{jt}-\frac{\Sigma_{\Delta,jt}^2}{2}\Big)\frac{\partial Q}{\partial\mathcal{K}_j}+\frac{1}{2}\sum_{j,\ell}\Sigma_{\Delta,j\ell t}\frac{\partial^2Q}{\partial\mathcal{K}_j\partial\mathcal{K}_\ell}&=\epsilon Q^k\Big(\mathrm{D}_{\mathcal{I}Q}\,q_{1t}+\sum_x\mathrm{D}_{\mathcal{I}x}\,x+\mathrm{d}_{\mathcal{I}T}\,\widehat{T}_t\Big),\end{aligned}$$

where

$$\mathrm{D}_{\mathcal{I}Q}\equiv\mathbf{diag}(g_{i,Q}^{ss}),\qquad\mathrm{D}_{\mathcal{I}R}\equiv\mathbf{diag}(g_{i,R}^{ss}),\qquad\mathrm{D}_{\mathcal{I}x}\equiv\mathrm{D}_{\mathcal{I}R}\,\mathrm{D}_{Rx}. \tag{C33}$$

Hence $\Theta^{k,x}=\mathrm{D}_{\mathcal{I}Q}Q^x+\mathrm{D}_{\mathcal{I}x}$, while $\Theta_t^{k,z}$ and $\Theta_t^{k,T}$ are unchanged.

Now, collecting the new derivations, we set $\mathbf{D}_r=0$ and $\mathbf{M}=\mathbf{diag}(M,0)$ in (C22). Moreover, the second row of $\mathbf{D}$ is $(\mathrm{D}_{QN},\mathrm{D}_{Q\mathcal{K}})$, and:

$$\mathbf{D}_{\mathcal{I}}=\begin{pmatrix}0&0\\ \mathrm{D}_{\mathcal{I}N}&\mathrm{D}_{\mathcal{I}\mathcal{K}}\end{pmatrix},\qquad\widetilde{\mathbf{M}}=\begin{pmatrix}M^*&0\\ \mathrm{D}_{\mathcal{I}N}&\mathrm{D}_{\mathcal{I}\mathcal{K}}\end{pmatrix},\qquad\mathbf{G}=\begin{pmatrix}G&0\\ 0&\mathrm{D}_{\mathcal{I}Q}\end{pmatrix}.$$

Given $\mathbf{V}$, the productivity and time blocks use:

$$\begin{aligned}\mathbf{D}^{\chi}&=\begin{pmatrix}\mathrm{D}_{u\chi}+V^k\mathrm{D}_{\mathcal{I}\chi}\\ \mathrm{D}_{Q\chi}+Q^k\mathrm{D}_{\mathcal{I}\chi}\end{pmatrix},\qquad\overline{\Psi}=\begin{pmatrix}M+V^nG&V^k\mathrm{D}_{\mathcal{I}Q}\\ Q^nG&Q^k\mathrm{D}_{\mathcal{I}Q}\end{pmatrix},\\ \Psi&=\begin{pmatrix}V^{\chi}\mathrm{d}_{XT}+V^k\mathrm{d}_{\mathcal{I}T}\\ \mathrm{d}_{QT}+Q^{\chi}\mathrm{d}_{XT}+Q^k\mathrm{d}_{\mathcal{I}T}\end{pmatrix},\end{aligned}$$

with $\overline{\Psi}=\mathbf{diag}(M,0)+\mathbf{VG}$. The four blocks have the same structure as in the diversified economy, with $\widetilde{\mathbf{I}}=\mathbf{I}$. This completes the proof of Proposition 4.

C.4 SAME: Proof of Proposition 5

We expand the Master Equations of Proposition 3 to second order and collect the coefficients of (16) and (17) to derive the SAME blocks of Proposition 5.

We first introduce a few conventions. To separate local risk from the temperature trend, we use a deterministic variable ς_t in place of T_t in the local variances of (5). At baseline, $\varsigma_t = T_t$, while the depreciation drift continues to depend on $\widehat{T}_t \equiv T_t + \sigma_t z_t$.

To keep the derivations as concise as possible, we omit the effect of baseline risk in our calculations and impose $\vartheta_{\Delta,i} = \vartheta_{\chi,i} = 0$ from the outset. This normalization is without loss of generality given our counterfactuals: we focus on changes in risk due to warming rather than evaluating the effect of baseline risk. Both risks are additive to second order and can be analyzed independently due to our superposition assumption in Appendix A. The covariance matrix of depreciation innovations is then $\Sigma_{\Delta,t} = \epsilon^2(\sigma_t z_t + \varsigma_t)\Sigma_\Delta$, where $\Sigma_{\Delta,ij} \equiv \Sigma_{\Delta,i}\Sigma_{\Delta,j}\mathfrak{c}_{\Delta,ij}$ and Σ_Δ^2 denotes the diagonal matrix of own variances. The same definitions apply to productivity shocks, replacing Δ with χ.

Throughout, we report the coefficients of $\epsilon^2/2$, grouped by the blocks (x, y), xz, xT, zz, TT, zT, $\sigma_t z$, ς_t, and the constant. The xz, xT, and zT coefficients carry a factor of 2 in (16); the blocks collect the brackets that multiply $2\sum_x[\,\cdot\,]xz$, $2\sum_x[\,\cdot\,]x$ and $2[\,\cdot\,]z$, that is, the coefficients without that factor. We symmetrize pair coefficients following Appendix C.1 and use the transition matrices of Appendix C.3.2. The trend counterparts follow by replacing $(v_t^z, q_t^z, \Theta_t^{s,z}, \sigma_t, \theta)$ with $(v_t^T, q_t^T, \Theta_t^{s,T}, T_t, 0)$. All tensor notation remains as before.

We derive the SAME for the diversified economy and present the modifications for the island economy at the end. Appendix H.1 provides the additional terms under Epstein-Zin preferences.

C.4.1 Second-Order Approximations

Workers' flow payoff. Workers' flow payoff depends on the state through the consumption index, which is log-linear in $(\chi_i, \mathcal{K}_i, \log N_i)$. Using Lemma 4, we obtain:

$$U_t - U^{ss} = \epsilon \sum_x \mathrm{D}_{Ux} x + \frac{\epsilon^2}{2} \sum_{x,y} \mathbb{D}_{Uxy} \times_3 y \times_2 x,$$

$$\mathbb{D}_{Uxy} = \mathbf{diag}_3\Big(\widetilde{C}_i^{ss}\big[u''(\widetilde{C}_i^{ss})\widetilde{C}_i^{ss} + u'(\widetilde{C}_i^{ss})\big]\mathrm{e}_{x,i}^C \mathrm{e}_{y,i}^C\Big) + \mathbb{1}_{x=y=n}\,\mathbf{diag}_3\Big(\frac{u'(\widetilde{C}_i^{ss})\widetilde{C}_i^{ss}\,\xi(1-\varpi)}{(N_i^{ss})^2}\Big),$$

where the loadings e^C are defined in Appendix C.3.

Capitalists' flow payoff. We first expand the rental rate $R_{it} = R_{0i} e^{\chi_{it}} K_{it}^{-\phi} N_{it}^{\psi}$ using Lemma 4:

$$R_t - R^{ss} = \epsilon \sum_x \mathrm{D}_{Rx} x + \frac{\epsilon^2}{2} \sum_{x,y} \mathbb{D}_{Rxy} \times_3 y \times_2 x,$$
$$\mathbb{D}_{Rxy} = \mathbf{diag}_3\big(R_i^{ss}\, \mathrm{e}^R_{x,i} \mathrm{e}^R_{y,i}\big) - \mathbb{1}_{x=y=n}\, \mathbf{diag}_3\Big(\frac{\psi R_i^{ss}}{(N_i^{ss})^2}\Big).$$

Including the continuation value from investment and the depreciation terms, we get:

$$W_t - W^{ss} = \epsilon\Big\{ \sum_x \mathrm{D}_{Qx} x + \mathrm{d}_{QT} \widehat{T}_t \Big\}$$
$$+ \frac{\epsilon^2}{2}\Big\{ \sum_{x,y} \mathbb{D}_{Rxy} \times_3 y \times_2 x + \mathrm{D}_{\mathcal{I}Q}(q_{1t} \cdot q_{1t}) + 2\, \mathrm{D}_{QQT}\, q_{1t}\, \widehat{T}_t \Big\},$$

where the first-order matrices are defined in (C15), and:

$$\mathrm{D}_{QQT} \equiv -\mathbf{diag}(\delta_i). \tag{C34}$$

Finally, applying Lemma 2 to the quadratic term $q_{1t} \cdot q_{1t}$ gives:

$$\begin{aligned} W_t - W^{ss} = \epsilon\Big\{ &\sum_x \mathrm{D}_{Qx} x + \mathrm{d}_{QT} \widehat{T}_t \Big\} \\ &+ \frac{\epsilon^2}{2}\Big\{ \sum_{x,y} \big[\mathbb{D}_{Rxy} + \mathbb{Q}^{x,y} \times_1 \mathrm{D}_{\mathcal{I}Q}\big] \times_3 y \times_2 x + 2 \sum_x \big[\mathrm{D}_{\mathcal{I}Q} \mathbf{diag}(q_t^z) Q^x + \mathrm{D}_{QQT} Q^x \sigma_t\big]\, x\, z \\ &\quad + 2 \sum_x \big[\mathrm{D}_{\mathcal{I}Q} \mathbf{diag}(q_t^T) Q^x + \mathrm{D}_{QQT} Q^x T_t\big]\, x + \big[\mathrm{D}_{\mathcal{I}Q}(q_t^z \cdot q_t^z) + 2\, \mathrm{D}_{QQT} q_t^z \sigma_t\big] z^2 \\ &\quad + \big[\mathrm{D}_{\mathcal{I}Q}(q_t^T \cdot q_t^T) + 2\, \mathrm{D}_{QQT} q_t^T T_t\big] + 2\big[\mathrm{D}_{\mathcal{I}Q}(q_t^z \cdot q_t^T) + \mathrm{D}_{QQT}(q_t^z T_t + q_t^T \sigma_t)\big] z \Big\}. \end{aligned}$$

Continuation value from migration. Let $M \equiv M(V^{ss})$ be the Jacobian defined in Appendix C.3, and let $H[\mathcal{M}_i]$ denote the Hessian of $\mathcal{M}_i$ evaluated at steady state. Expanding the migration continuation value to second order gives:

$$\mathcal{M}_i(V_t) - \mathcal{M}_i(V^{ss}) = \epsilon [M v_1]_i + \frac{\epsilon^2}{2}\Big\{ [M v_2]_i + v_1^* H[\mathcal{M}_i] v_1 \Big\},$$
$$H[\mathcal{M}_i](V^{ss}) = \Big[\mu \frac{\partial m_{ik}}{\partial V_h}\Big]_{hk}, \qquad v_1^* H[\mathcal{M}_i] v_1 = \mu\nu \Big[\sum_k m_{ik} v_{1k}^2 - \sum_h \sum_k m_{ih} m_{ik} v_{1h} v_{1k} \Big],$$

where m_{ik} are the steady-state migration shares. We can express the quadratic term in tensor notation as $\mu\nu[m(v_1 \cdot v_1) + \mathbb{M} \times_3 v_1 \times_2 v_1]$, where $\mathbb{M}_{ikh} \equiv -m_{ik} m_{ih}$ is symmetric in its last two indices. Substituting (16) for v_1 and v_2, Lemma 1 writes Mv_2 as $\mathcal{V}^{x,y} \times_1 M$ on the pairs, and

Lemma 2 expands the quadratic form with the outer products $\mathbb{V}^{x,y}$ of Appendix C.1. Collecting the terms gives:

$$\begin{aligned}
\mathcal{M}(V_t) - \mathcal{M}(V^{ss}) = \epsilon\Big\{ & \sum_x MV^x\, x + Mv_t^z\, z + Mv_t^T \Big\} \\
& + \frac{\epsilon^2}{2}\Big\{ \sum_{x,y} \big[\mathcal{V}^{x,y} \times_1 M + \mu\nu\big(\mathbb{V}^{x,y} \times_1 m + \mathbb{M} \times_3 V^y \times_2 V^x\big)\big] \times_3 y \times_2 x \\
& \quad + 2\sum_x \big[MV_t^{x,z} + \mu\nu\big(m\mathbf{diag}(v_t^z)V^x + \mathbb{M} \times_2 V^x \times_3 v_t^z\big)\big]\, x\, z \\
& \quad + 2\sum_x \big[MV_t^{x,T} + \mu\nu\big(m\mathbf{diag}(v_t^T)V^x + \mathbb{M} \times_2 V^x \times_3 v_t^T\big)\big]\, x \\
& \quad + \big[Mv_t^{z,z} + \mu\nu\big(m(v_t^z \cdot v_t^z) + \mathbb{M} \times_3 v_t^z \times_2 v_t^z\big)\big] z^2 \\
& \quad + \big[Mv_t^{T,T} + \mu\nu\big(m(v_t^T \cdot v_t^T) + \mathbb{M} \times_3 v_t^T \times_2 v_t^T\big)\big] \\
& \quad + 2\big[Mv_t^{z,T} + \mu\nu\big(m(v_t^z \cdot v_t^T) + \mathbb{M} \times_3 v_t^z \times_2 v_t^T\big)\big] z \\
& \quad + Mv_t^{\Sigma,z}\, z + Mv_t^{\Sigma,T} + M\overline{v}_t \Big\}.
\end{aligned}$$

Continuation value from aggregate shocks. The continuation value from the aggregate shocks involves only the objects that depend on z. Up to second order, it is given by

$$\begin{aligned}
\mathcal{A}(z) \quad & \Big[\quad \epsilon v_t^z z + \frac{\epsilon^2}{2}\Big(2\sum_x V_t^{x,z}\, x\, z + v_t^{z,z} z^2 + 2v_t^{z,T} z + v_t^{\Sigma,z} z\Big)\Big] \\
= \quad & -\epsilon\theta v_t^z z + \frac{\epsilon^2}{2}\Big\{ -2\theta \sum_x V_t^{x,z}\, x\, z - 2\theta v_t^{z,z} z^2 - 2\theta v_t^{z,T} z - \theta v_t^{\Sigma,z} z + v_t^{z,z} \Big\}, \\
\mathcal{A}(z) \quad & \Big[\quad \epsilon q_t^z z + \frac{\epsilon^2}{2}\Big(2\sum_x Q_t^{x,z}\, x\, z + q_t^{z,z} z^2 + 2q_t^{z,T} z + q_t^{\Sigma,z} z\Big)\Big] \\
= \quad & -\epsilon\theta q_t^z z + \frac{\epsilon^2}{2}\Big\{ -2\theta \sum_x Q_t^{x,z}\, x\, z - 2\theta q_t^{z,z} z^2 - 2\theta q_t^{z,T} z - \theta q_t^{\Sigma,z} z + q_t^{z,z} \Big\}.
\end{aligned}$$

Continuation value from changes in the productivity distribution. First, denote by $\mu_{1t}^{\chi} = -\kappa\widehat{\chi} + \mathrm{d}_{XT}\widehat{T}_t = \sum_x \Theta^{\chi,x} x + \Theta_t^{\chi,z} z + \Theta_t^{\chi,T}$ the first-order drift of productivity, where d_{XT} is defined in (C16). Since this drift is linear, it has no second-order component. The first-order contribution to the continuation value is $\epsilon V^{\chi}\mu_{1t}^{\chi}$, while the second-order terms follow from Lemma 3 with $s = \chi$. The Itô correction uses $\partial^2_{\chi_j\chi_\ell} V_i = \mathcal{V}_{ij\ell}^{\chi,\chi} + \mathcal{O}(\epsilon)$ and the contraction defined in Appendix C.1. For

workers, we obtain:

$$\begin{aligned}
&\sum_j (X_{jt} - \kappa\chi_j)\frac{\partial V}{\partial \chi_j} + \frac{1}{2}\sum_{j,\ell}\Sigma_{\chi,j\ell t}\frac{\partial^2 V}{\partial \chi_j \partial \chi_\ell} \\
&= \epsilon V^{\chi}\big(-\kappa\,\widehat{\chi} + \mathrm{d}_{XT}\,\widehat{T}_t\big) \\
&\quad + \frac{\epsilon^2}{2}\Big\{ -\kappa\sum_{x,y}\big[\mathbb{1}_{x=\chi}\mathcal{V}^{\chi,y} + \mathbb{1}_{y=\chi}\mathcal{V}^{x,\chi}\big]\times_3 y \times_2 x \\
&\qquad + 2\sum_x\Big[\tfrac{1}{2}\sigma_t\big(\mathcal{V}^{\chi,x}\times_2 \mathrm{d}_{XT} + \mathcal{V}^{x,\chi}\times_3 \mathrm{d}_{XT}\big) - \kappa\mathbb{1}_{x=\chi}V_t^{\chi,z}\Big]x\,z \\
&\qquad + 2\sum_x\Big[\tfrac{1}{2}T_t\big(\mathcal{V}^{\chi,x}\times_2 \mathrm{d}_{XT} + \mathcal{V}^{x,\chi}\times_3 \mathrm{d}_{XT}\big) - \kappa\mathbb{1}_{x=\chi}V_t^{\chi,T}\Big]x \\
&\qquad + 2\,\sigma_t V_t^{\chi,z}\mathrm{d}_{XT}\,z^2 + 2\,T_t V_t^{\chi,T}\mathrm{d}_{XT} + 2\big[T_t V_t^{\chi,z} + \sigma_t V_t^{\chi,T}\big]\mathrm{d}_{XT}\,z \\
&\qquad + (\sigma_t z + \varsigma_t)\,\mathcal{V}^{\chi,\chi}\times_{2,3}\Sigma_\chi\Big\},
\end{aligned}$$

and for capitalists:

$$\begin{aligned}
&\sum_j (X_{jt} - \kappa\chi_j)\frac{\partial Q}{\partial \chi_j} + \frac{1}{2}\sum_{j,\ell}\Sigma_{\chi,j\ell t}\frac{\partial^2 Q}{\partial \chi_j \partial \chi_\ell} \\
&= \epsilon Q^{\chi}\big(-\kappa\,\widehat{\chi} + \mathrm{d}_{XT}\,\widehat{T}_t\big) \\
&\quad + \frac{\epsilon^2}{2}\Big\{ -\kappa\sum_{x,y}\big[\mathbb{1}_{x=\chi}\mathcal{Q}^{\chi,y} + \mathbb{1}_{y=\chi}\mathcal{Q}^{x,\chi}\big]\times_3 y \times_2 x \\
&\qquad + 2\sum_x\Big[\tfrac{1}{2}\sigma_t\big(\mathcal{Q}^{\chi,x}\times_2 \mathrm{d}_{XT} + \mathcal{Q}^{x,\chi}\times_3 \mathrm{d}_{XT}\big) - \kappa\mathbb{1}_{x=\chi}Q_t^{\chi,z}\Big]x\,z \\
&\qquad + 2\sum_x\Big[\tfrac{1}{2}T_t\big(\mathcal{Q}^{\chi,x}\times_2 \mathrm{d}_{XT} + \mathcal{Q}^{x,\chi}\times_3 \mathrm{d}_{XT}\big) - \kappa\mathbb{1}_{x=\chi}Q_t^{\chi,T}\Big]x \\
&\qquad + 2\,\sigma_t Q_t^{\chi,z}\mathrm{d}_{XT}\,z^2 + 2\,T_t Q_t^{\chi,T}\mathrm{d}_{XT} + 2\big[T_t Q_t^{\chi,z} + \sigma_t Q_t^{\chi,T}\big]\mathrm{d}_{XT}\,z \\
&\qquad + (\sigma_t z + \varsigma_t)\,\mathcal{Q}^{\chi,\chi}\times_{2,3}\Sigma_\chi\Big\}.
\end{aligned}$$

Continuation value from changes in the population distribution. Changes in population affect continuation values through the migration flow $M^*(V)N$. For workers, this contribution is $\sum_j \partial_{N_j} V_i\,(M^*(V)N)_j$, with Q_i replacing V_i for capitalists. The first derivative of $M^*_{jk}(V) = \mu\big(m_{kj}(V) - \mathbb{1}_{j=k}\big)$ is:

$$\frac{\partial M^*_{jk}}{\partial V_h} = \mu\nu\begin{cases} -m_{kj}m_{kh} & \text{if } j \neq h, \\ m_{kj}(1-m_{kj}) & \text{if } j = h, \end{cases} \equiv \mathbb{G}_{jkh}.$$

The second derivative is the four-dimensional tensor $\mathbb{H}_{jkhs} \equiv \mu\, \partial^2 m_{kj}/\partial V_h \partial V_s$, where j indexes destinations and k origins. We use the tensor contractions of Appendix C.1, so that $\mathbb{H} \times_2 N$ has entries $\sum_k \mathbb{H}_{jkhs} N_k$. The Hessian entries are:

$$\frac{\partial^2 m_{kj}}{\partial V_h^2} = \nu^2 m_{kj} m_{kh}(2m_{kh} - 1), \qquad \frac{\partial^2 m_{kj}}{\partial V_h \partial V_s} = 2\nu^2 m_{kj} m_{kh} m_{ks},$$
$$\frac{\partial^2 m_{kj}}{\partial V_h \partial V_j} = \nu^2 m_{kj} m_{kh}(2m_{kj} - 1), \qquad \frac{\partial^2 m_{kj}}{\partial V_j^2} = \nu^2 m_{kj}(1 - 3m_{kj} + 2m_{kj}^2),$$

corresponding to $h \neq j$; $h \neq s$ with $h, s \neq j$; $h \neq j$ with one derivative taken with respect to V_j; and both derivatives with respect to V_j, respectively.

Expanding the migration flow $M^*(V)N$ around steady state, with $G = \mathbb{G} \times_2 N$ as in (C18), gives the population drift:

$$\mu_t^n = \mu_{1t}^n + \epsilon\, \mu_{2t}^n, \qquad \mu_{1t}^n = M^* n + G v_1 = \sum_x \Theta^{n,x} x + \Theta_t^{n,z} z + \Theta_t^{n,T},$$
$$\mu_{2t}^n = \tfrac{1}{2} G v_2 + \mathbb{G} \times_3 v_1 \times_2 n + \tfrac{1}{2}(\mathbb{H} \times_2 N) \times_3 v_1 \times_2 v_1. \tag{C35}$$

Using $\partial_{N_j} V_i = V_{ij}^n + \frac{\epsilon}{2} \partial_{n_j} v_{2,i}$, the continuation value becomes, to second order:

$$\sum_j \frac{\partial V_i}{\partial N_j} \epsilon \mu_{jt}^n = \epsilon V^n \mu_{1t}^n + \frac{\epsilon^2}{2}\Big[2V^n \mu_{2t}^n + (\partial_n v_2)\, \mu_{1t}^n\Big].$$

The second term in brackets follows from Lemma 3 with $s = n$. To express the first term more compactly, define:

$$\mathrm{X}_W \equiv \mathbb{G} \times_1 W^n \times_2 N = W^n G, \qquad \mathbb{Y}_W \equiv \mathbb{G} \times_1 W^n, \qquad \mathbb{Z}_W \equiv \mathbb{H} \times_1 W^n \times_2 N,$$

for $W \in \{V, Q\}$. By Lemma 1, we can write $2V^n \mu_{2t}^n = \mathrm{X}_V v_2 + 2\,\mathbb{Y}_V \times_3 v_1 \times_2 n + \mathbb{Z}_V \times_3 v_1 \times_2 v_1$. We expand the quadratic term using Lemma 2 and symmetrize the mixed pairs involving n.

Substituting the expansion (16), the worker continuation value becomes:

$$
\begin{aligned}
&\sum_j \big(M^*(V_t)N_t\big)_j \frac{\partial V}{\partial N_j} \\
&\quad = \epsilon V^n \big(M^* n + G v_{1t}\big) \\
&\qquad + \frac{\epsilon^2}{2}\Big\{ \sum_{x,y} \Big[\mathcal{V}^{x,y} \times_1 \mathrm{X}_V + \mathcal{V}^{n,y} \times_2 \Theta^{n,x} + \mathcal{V}^{x,n} \times_3 \Theta^{n,y} \\
&\qquad\qquad + \mathbb{1}_{x=n} \mathbb{Y}_V \times_3 V^y + \mathbb{1}_{y=n} \mathbb{Y}^*_V \times_2 V^x + \mathbb{Z}_V \times_3 V^y \times_2 V^x \Big] \times_3 y \times_2 x \\
&\qquad + 2 \sum_x \Big[\mathrm{X}_V V_t^{x,z} + \tfrac{1}{2}\big(\mathcal{V}^{n,x} \times_2 \Theta_t^{n,z} + \mathcal{V}^{x,n} \times_3 \Theta_t^{n,z}\big) + V_t^{n,z}\Theta^{n,x} \\
&\qquad\qquad + \mathbb{1}_{x=n} \mathbb{Y}_V \times_3 v_t^z + \mathbb{Z}_V \times_2 V^x \times_3 v_t^z \Big] x\, z \\
&\qquad + 2 \sum_x \Big[\mathrm{X}_V V_t^{x,T} + \tfrac{1}{2}\big(\mathcal{V}^{n,x} \times_2 \Theta_t^{n,T} + \mathcal{V}^{x,n} \times_3 \Theta_t^{n,T}\big) + V_t^{n,T}\Theta^{n,x} \\
&\qquad\qquad + \mathbb{1}_{x=n} \mathbb{Y}_V \times_3 v_t^T + \mathbb{Z}_V \times_2 V^x \times_3 v_t^T \Big] x \\
&\qquad + \big[\mathrm{X}_V v_t^{z,z} + 2 V_t^{n,z}\Theta_t^{n,z} + \mathbb{Z}_V \times_3 v_t^z \times_2 v_t^z \big] z^2 \\
&\qquad + \big[\mathrm{X}_V v_t^{T,T} + 2 V_t^{n,T}\Theta_t^{n,T} + \mathbb{Z}_V \times_3 v_t^T \times_2 v_t^T \big] \\
&\qquad + 2 \big[\mathrm{X}_V v_t^{z,T} + V_t^{n,z}\Theta_t^{n,T} + V_t^{n,T}\Theta_t^{n,z} + \mathbb{Z}_V \times_3 v_t^z \times_2 v_t^T \big] z \\
&\qquad + \mathrm{X}_V v_t^{\Sigma,z} z + \mathrm{X}_V v_t^{\Sigma,T} + \mathrm{X}_V \overline{v}_t \Big\}.
\end{aligned}
$$

This expression separates the unknown second-order coefficients from the products of first-order coefficients already determined by the FAME.

For capitalists, we replace V^n with Q^n in X, $\mathbb{Y}$, and $\mathbb{Z}$, and the derivatives $\mathcal{V}^{n,y}$, $\mathcal{V}^{x,n}$, $V_t^{n,z}$, and $V_t^{n,T}$ with their corresponding Q counterparts. All remaining terms continue to depend on workers' values, since the population drift is determined by migration decisions. In particular, X_Q multiplies the worker second-order coefficients v_{2t}, while $\mathbb{Y}_Q$ and $\mathbb{Z}_Q$ involve workers' first-order responses.

Continuation value from changes in the capital distribution. Changes in the capital distribution affect continuation values through the drift of log capital. Writing $\mu_t^k = \mu_{1t}^k + \epsilon \mu_{2t}^k$, the first- and second-order drifts are:

$$
\begin{aligned}
\mu_{1t}^k &= \mathrm{D}_{\mathcal{I}Q} q_{1t} + \mathrm{d}_{\mathcal{I}T} \widehat{T}_t = \sum_x \Theta^{k,x} x + \Theta_t^{k,z} z + \Theta_t^{k,T}, \\
\mu_{2t}^k &= \tfrac{1}{2}\Big[\mathrm{D}_{\mathcal{I}Q} q_{2t} + \mathrm{D}_{\mathcal{I}QQ}(q_{1t} \cdot q_{1t}) + \Sigma_\Delta^2 \mathrm{d}_{\mathcal{I}\Sigma T}(\sigma_t z + \varsigma_t) \Big],
\end{aligned}
\tag{C36}
$$

where $\mathrm{D}_{\mathcal{I}Q}$ and $\mathrm{d}_{\mathcal{I}T}$ are defined in (C17), and:

$$\mathrm{D}_{\mathcal{I}QQ} \equiv \zeta(\zeta-1)\,\mathbf{diag}\Big(\frac{\Delta_i}{(Q_i^{ss})^2}\Big), \qquad \mathrm{d}_{\mathcal{I}\Sigma T} \equiv -\mathbb{1}.$$

The last term in μ_{2t}^k comes from the Itô correction $-\Sigma_{\Delta,it}^2/2$ in the log-capital drift.

As in the population case, the continuation value expands as $\epsilon V^k \mu_{1t}^k + \frac{\epsilon^2}{2}\big[2V^k\mu_{2t}^k + (\partial_k v_2)\mu_{1t}^k\big]$. To simplify the second-order terms, define:

$$A_W \equiv W^k \mathrm{D}_{\mathcal{I}Q}, \qquad B_W \equiv W^k \mathrm{D}_{\mathcal{I}QQ}, \qquad W \in \{V, Q\}.$$

We can then write $2V^k\mu_{2t}^k = A_V q_2 + B_V(q_{1t}\cdot q_{1t}) + V^k\Sigma_\Delta^2 \mathrm{d}_{\mathcal{I}\Sigma T}(\sigma_t z + \varsigma_t)$. We expand these terms using Lemmas 2 and 1, while $(\partial_k v_2)\mu_{1t}^k$ follows from Lemma 3 with $s = k$. The generator's Itô correction uses $\partial^2_{\mathcal{K}_j\mathcal{K}_\ell} V_i = \mathcal{V}_{ij\ell}^{k,k} + \mathcal{O}(\epsilon)$ and the tensor contraction of Appendix C.1. Collecting these contributions, we obtain for workers:

$$\begin{aligned}
&\sum_j \Big(c_{j0}Q_{jt}^\zeta - \Delta_{jt} - \frac{\Sigma_{\Delta,jt}^2}{2}\Big)\frac{\partial V}{\partial \mathcal{K}_j} + \frac{1}{2}\sum_{j,\ell}\Sigma_{\Delta,j\ell t}\frac{\partial^2 V}{\partial\mathcal{K}_j\partial\mathcal{K}_\ell} \\
&= \epsilon V^k\big(\mathrm{D}_{\mathcal{I}Q}q_{1t} + \mathrm{d}_{\mathcal{I}T}\widehat{T}_t\big) \\
&\quad + \frac{\epsilon^2}{2}\Big\{\sum_{x,y}\Big[\mathcal{Q}^{x,y}\times_1 A_V + \mathcal{V}^{k,y}\times_2\Theta^{k,x} + \mathcal{V}^{x,k}\times_3\Theta^{k,y} \\
&\qquad\quad + \mathbb{Q}^{x,y}\times_1 B_V\Big]\times_3 y\times_2 x \\
&\qquad + 2\sum_x\Big[A_V Q_t^{x,z} + \tfrac{1}{2}\big(\mathcal{V}^{k,x}\times_2\Theta_t^{k,z} + \mathcal{V}^{x,k}\times_3\Theta_t^{k,z}\big) + V_t^{k,z}\Theta^{k,x} \\
&\qquad\quad + B_V\mathbf{diag}(q_t^z)Q^x\Big]x\,z \\
&\qquad + 2\sum_x\Big[A_V Q_t^{x,T} + \tfrac{1}{2}\big(\mathcal{V}^{k,x}\times_2\Theta_t^{k,T} + \mathcal{V}^{x,k}\times_3\Theta_t^{k,T}\big) + V_t^{k,T}\Theta^{k,x} \\
&\qquad\quad + B_V\mathbf{diag}(q_t^T)Q^x\Big]x \\
&\qquad + \big[A_V q_t^{z,z} + 2V_t^{k,z}\Theta_t^{k,z} + B_V(q_t^z\cdot q_t^z)\big]z^2 \\
&\qquad + \big[A_V q_t^{T,T} + 2V_t^{k,T}\Theta_t^{k,T} + B_V(q_t^T\cdot q_t^T)\big] \\
&\qquad + 2\big[A_V q_t^{z,T} + V_t^{k,z}\Theta_t^{k,T} + V_t^{k,T}\Theta_t^{k,z} + B_V(q_t^z\cdot q_t^T)\big]z \\
&\qquad + A_V q_t^{\Sigma,z}\,z + A_V q_t^{\Sigma,T} + A_V\bar{q}_t + \Big[V^k\Sigma_\Delta^2\mathrm{d}_{\mathcal{I}\Sigma T} + \mathcal{V}^{k,k}\times_{2,3}\Sigma_\Delta\Big](\sigma_t z + \varsigma_t)\Big\}.
\end{aligned}$$

For capitalists, the same expression applies with Q replacing V: we use A_Q, B_Q, Q^k, and the corresponding $\mathcal{Q}$ objects from Lemma 3. The final bracket combines the Itô corrections from the

capital drift and the generator, both of which enter the Σ blocks.

Risk-free rate (diversified model only). We now expand the interest rate adjustment $\mathcal{R}_t$ of (C11). Writing $\mathcal{R}_t = -\frac{\overline{\gamma}}{\mathcal{C}_t}\mathcal{S}_t Q_t$, we define the scalar drift sum:

$$\mathcal{S}_t \equiv \sum_{x\in\{\chi,N,\mathcal{K}\}} \sum_j \mathcal{C}_{j,x}\,\mu_j^x + \sum_j \mathcal{C}_{j,Q}\,\mathcal{L}_t Q_j, \tag{C37}$$

where $\mathcal{C}_{j,x}$ are the derivatives of local capitalist consumption defined in Appendix C.2. We expand $\mathcal{S}_t = \epsilon\mathcal{S}_{1t} + \frac{\epsilon^2}{2}\mathcal{S}_{2t}$, $\mathcal{C}_t = C^{ss} + \epsilon C_{1t}$, and $\mathcal{L}_t Q = \epsilon(\mathcal{L}_t Q)_1 + \frac{\epsilon^2}{2}(\mathcal{L}_t Q)_2$, where $(\mathcal{L}_t Q)_2$ collects the second-order continuation terms of the capitalist equation. Substituting these expansions gives, to second order:

$$\mathcal{R}_t = -\epsilon\,\frac{\overline{\gamma}}{C^{ss}}\,\mathcal{S}_{1t}\,Q^{ss} - \frac{\epsilon^2}{2}\,\frac{\overline{\gamma}}{C^{ss}}\Big[2\mathcal{S}_{1t}\,q_{1t} - \frac{2C_{1t}}{C^{ss}}\,\mathcal{S}_{1t}\,Q^{ss} + \mathcal{S}_{2t}\,Q^{ss}\Big]. \tag{C38}$$

The first term recovers the FAME adjustment in (C19). The second-order correction has three components: products of first-order terms, second-order drifts weighted by steady-state consumption derivatives, and changes in these derivatives multiplied by first-order drifts. We separate the last two as $\mathcal{S}_{2t} = \mathcal{S}_{2t}^{0\times 2} + \mathcal{S}_{2t}^{1\times 1}$.

We first collect the objects already determined by the FAME. Using the consumption derivatives in (C20), the first-order consumption deviation is:

$$C_{1t} = \sum_x \mathcal{C}_x x + \mathcal{C}_Q q_{1t}. \tag{C39}$$

Similarly, the first-order drift sum is:

$$\begin{aligned}\frac{\overline{\gamma}}{C^{ss}}\,\mathcal{S}_{1t} &= \sum_x \varrho^x x + \varrho_t^z z + \varrho_t^T,\\ \varrho^x &\equiv \frac{\overline{\gamma}}{C^{ss}}\Big[\sum_s \mathcal{C}_s\Theta^{s,x} + \mathcal{C}_Q\mu^{Q,x}\Big], \qquad \varrho_t^z \equiv \frac{\overline{\gamma}}{C^{ss}}\Big[\sum_s \mathcal{C}_s\Theta_t^{s,z} + \mathcal{C}_Q\mu_t^{Q,z}\Big],\end{aligned} \tag{C40}$$

where $\mathcal{C}_s$ denotes the corresponding consumption derivative for $s \in \{n,k,\chi\}$. The first-order capital-price drift loadings are:

$$\mu^{Q,x} \equiv \sum_s Q^s\Theta^{s,x}, \qquad \mu_t^{Q,z} \equiv \sum_s Q^s\Theta_t^{s,z} + \frac{\partial q_t^z}{\partial t} - \theta q_t^z, \qquad \mu_t^{Q,T} \equiv \sum_s Q^s\Theta_t^{s,T} + \frac{\partial q_t^T}{\partial t},$$

so that $(\mathcal{L}_t Q)_1 = \sum_x \mu^{Q,x} x + \mu_t^{Q,z} z + \mu_t^{Q,T}$. The loading ϱ_t^T follows from the corresponding T

substitution.

We now turn to the cross terms in (C38). These involve only known first-order objects, $\frac{\overline{\gamma}}{C^{ss}}\left[2\mathcal{S}_{1t}q_{1t} - \frac{2C_{1t}}{C^{ss}}\mathcal{S}_{1t}Q^{ss}\right]$. Expanding their products and collecting the coefficients of (17), we obtain:

$$\begin{aligned}
\mathbb{J}^{x,y}_{ijh} &\equiv Q^x_{ij}\varrho^y_h + Q^y_{ih}\varrho^x_j - \frac{Q^{ss}_i}{C^{ss}}\Big[\left(\mathcal{C}_x + \mathcal{C}_Q Q^x\right)_j \varrho^y_h + \left(\mathcal{C}_y + \mathcal{C}_Q Q^y\right)_h \varrho^x_j\Big],\\
\mathrm{J}^{x,z}_t &\equiv q^z_t\,\varrho^x + \varrho^z_t\,Q^x - \frac{1}{C^{ss}}\,Q^{ss}\Big[\mathcal{C}_Q q^z_t\,\varrho^x + \varrho^z_t\left(\mathcal{C}_x + \mathcal{C}_Q Q^x\right)\Big],\\
\mathrm{j}^{zz}_t &\equiv 2\varrho^z_t\Big[q^z_t - \frac{\mathcal{C}_Q q^z_t}{C^{ss}}Q^{ss}\Big], \qquad \mathrm{j}^{TT}_t \equiv 2\varrho^T_t\Big[q^T_t - \frac{\mathcal{C}_Q q^T_t}{C^{ss}}Q^{ss}\Big],\\
\mathrm{j}^{zT}_t &\equiv \varrho^z_t\Big[q^T_t - \frac{\mathcal{C}_Q q^T_t}{C^{ss}}Q^{ss}\Big] + \varrho^T_t\Big[q^z_t - \frac{\mathcal{C}_Q q^z_t}{C^{ss}}Q^{ss}\Big].
\end{aligned} \tag{C41}$$

Here and in the expressions below, $Q^{ss}[\,\cdot\,]$ denotes the outer product of the column Q^{ss} with a row, as in (C20).

We next consider the second-order drifts weighted by steady-state consumption derivatives. We group the Itô correction $-\Sigma^2_\Delta/2$ of the capital drift with the consumption Itô terms, so $\mathcal{S}^{0\times 2}_{2t}$ contains only the price-dependent part of the capital drift. Since mean depreciation and productivity drifts are linear in $\widehat{T}_t$, there are no second-order temperature terms. Using (C35) and (C36), we obtain:

$$\begin{aligned}
\mathcal{S}^{0\times 2}_{2t} \equiv\; & \mathcal{C}_n\Big[G v_{2t} + 2\,\mathbb{G}\times_3 v_{1t}\times_2 n + (\mathbb{H}\times_2 N)\times_3 v_{1t}\times_2 v_{1t}\Big]\\
& + \mathcal{C}_k\Big[\mathrm{D}_{\mathcal{I}Q} q_{2t} + \mathrm{D}_{\mathcal{I}QQ}(q_{1t}\cdot q_{1t})\Big] + \mathcal{C}_Q(\mathcal{L}_t Q)_2.
\end{aligned} \tag{C42}$$

This expression contains the second-order unknowns v_{2t}, q_{2t}, and $(\mathcal{L}_t Q)_2$, together with known products of first-order coefficients. Using the interest rate matrices in (C20), we separate these contributions as $\frac{\overline{\gamma}}{C^{ss}}\mathcal{S}^{0\times 2}_{2t}Q^{ss} = \mathrm{D}_{rV}v_{2t} + \mathrm{D}_{rQ}q_{2t} + \mathrm{D}_{r\mu}(\mathcal{L}_t Q)_2 + \mathcal{R}^{0\times 2}_{2t}$. The known coefficients of $\mathcal{R}^{0\times 2}_{2t}$ are:

$$\begin{aligned}
\mathbb{K}^{x,y}_{ijh} &\equiv \frac{\overline{\gamma}}{C^{ss}}\,Q^{ss}_i\Big[\mathbb{1}_{x=n}Y_{\mathcal{C}}V^y + \mathbb{1}_{y=n}\big(Y_{\mathcal{C}}V^x\big)^* + (V^x)^* Z_{\mathcal{C}}V^y + (Q^x)^*\mathbf{diag}\big(\mathcal{C}_k\mathrm{D}_{\mathcal{I}QQ}\big)Q^y\Big]_{jh},\\
\mathrm{K}^{x,z}_t &\equiv \frac{\overline{\gamma}}{C^{ss}}\,Q^{ss}\Big[\mathbb{1}_{x=n}Y_{\mathcal{C}}v^z_t + (V^x)^* Z_{\mathcal{C}}v^z_t + (Q^x)^*\mathbf{diag}\big(\mathcal{C}_k\mathrm{D}_{\mathcal{I}QQ}\big)q^z_t\Big]^*,\\
\mathrm{k}^{zz}_t &\equiv \frac{\overline{\gamma}}{C^{ss}}\Big[(v^z_t)^* Z_{\mathcal{C}}v^z_t + (q^z_t)^*\mathbf{diag}\big(\mathcal{C}_k\mathrm{D}_{\mathcal{I}QQ}\big)q^z_t\Big]Q^{ss},\\
\mathrm{k}^{zT}_t &\equiv \frac{\overline{\gamma}}{C^{ss}}\Big[(v^z_t)^* Z_{\mathcal{C}}v^T_t + (q^z_t)^*\mathbf{diag}\big(\mathcal{C}_k\mathrm{D}_{\mathcal{I}QQ}\big)q^T_t\Big]Q^{ss},
\end{aligned} \tag{C43}$$

where $Y_{\mathcal{C}} \equiv \mathbb{G}\times_1 \mathcal{C}_n$ and $Z_{\mathcal{C}} \equiv \mathbb{H}\times_1 \mathcal{C}_n \times_2 N$. The coefficient k^{TT}_t follows from the T substitution.

We now turn to the changes in consumption derivatives that multiply first-order drifts. These depend on the following second derivatives of aggregate consumption:

$$\begin{aligned}
&\mathcal{C}_{\chi\chi} \equiv \mathbf{vec}\big(R_i^{ss}K_i^{ss}\big), \qquad \mathcal{C}_{\chi k} \equiv (1-\phi)\,\mathcal{C}_{\chi\chi}, \qquad \mathcal{C}_{\chi n} \equiv \psi\,\mathbf{vec}\big(R_i^{ss}K_i^{ss}/N_i^{ss}\big),\\
&\mathcal{C}_{kk} \equiv (1-\phi)^2\mathcal{C}_{\chi\chi} - \tfrac{\zeta}{1+\zeta}\mathbf{vec}\big(\Delta_i Q_i^{ss}K_i^{ss}\big), \quad \mathcal{C}_{kn} \equiv (1-\phi)\,\mathcal{C}_{\chi n}, \quad \mathcal{C}_{kQ} \equiv -\zeta\,\mathbf{vec}\big(\Delta_i K_i^{ss}\big),\\
&\mathcal{C}_{nn} \equiv (\psi-1)\,\mathbf{vec}\big(\psi R_i^{ss}K_i^{ss}/(N_i^{ss})^2\big), \qquad \mathcal{C}_{QQ} \equiv -\zeta^2\,\mathbf{vec}\big(\Delta_i K_i^{ss}/Q_i^{ss}\big),
\end{aligned} \tag{C44}$$

where k denotes log capital and $\mathcal{C}_{\chi Q} = \mathcal{C}_{nQ} = 0$. For example, the deviation of the row $\mathcal{C}_\chi$ is $(\mathcal{C}_{\chi\chi}\cdot\widehat{\chi} + \mathcal{C}_{\chi k}\cdot k + \mathcal{C}_{\chi n}\cdot n)^*$. The remaining rows follow analogously, with $\mathcal{C}_{kQ}\cdot q_{1t}$ and $\mathcal{C}_{QQ}\cdot q_{1t}$ entering $\mathcal{C}_k$ and $\mathcal{C}_Q$, respectively.

Multiplying these row deviations by their corresponding first-order state drifts gives $\mathcal{S}_{2t}^{1\times 1}$. To collect the coefficients, we define the row loadings for each state x:

$$\begin{aligned}
\Gamma_\chi^x &\equiv \mathbf{diag}(\mathcal{C}_{\chi\chi})\mathbb{1}_{x=\chi} + \mathbf{diag}(\mathcal{C}_{\chi k})\mathbb{1}_{x=k} + \mathbf{diag}(\mathcal{C}_{\chi n})\mathbb{1}_{x=n},\\
\Gamma_k^x &\equiv \mathbf{diag}(\mathcal{C}_{\chi k})\mathbb{1}_{x=\chi} + \mathbf{diag}(\mathcal{C}_{kk})\mathbb{1}_{x=k} + \mathbf{diag}(\mathcal{C}_{kn})\mathbb{1}_{x=n} + \mathbf{diag}(\mathcal{C}_{kQ})Q^x,\\
\Gamma_n^x &\equiv \mathbf{diag}(\mathcal{C}_{\chi n})\mathbb{1}_{x=\chi} + \mathbf{diag}(\mathcal{C}_{kn})\mathbb{1}_{x=k} + \mathbf{diag}(\mathcal{C}_{nn})\mathbb{1}_{x=n},\\
\Gamma_Q^x &\equiv \mathbf{diag}(\mathcal{C}_{kQ})\mathbb{1}_{x=k} + \mathbf{diag}(\mathcal{C}_{QQ})Q^x.
\end{aligned} \tag{C45}$$

Only the rows $\mathcal{C}_k$ and $\mathcal{C}_Q$ have time-dependent loadings, through q_{1t}. Their z and constant loadings are $\mathbf{diag}(\mathcal{C}_{kQ})q_t^z$, $\mathbf{diag}(\mathcal{C}_{kQ})q_t^T$, $\mathbf{diag}(\mathcal{C}_{QQ})q_t^z$, and $\mathbf{diag}(\mathcal{C}_{QQ})q_t^T$, respectively.

Defining $\mathrm{H}^{x,y} \equiv \sum_s(\Gamma_s^x)^*\Theta^{s,y} + (\Gamma_Q^x)^*\mu^{Q,y}$, the coefficients of $\frac{\overline{\gamma}}{C^{ss}}\mathcal{S}_{2t}^{1\times 1}Q^{ss}$ are:

$$\begin{aligned}
\mathbb{L}_{ijh}^{x,y} &\equiv \frac{\overline{\gamma}}{C^{ss}}\,Q_i^{ss}\Big[\mathrm{H}^{x,y} + \big(\mathrm{H}^{y,x}\big)^*\Big]_{jh},\\
\mathrm{L}_t^{x,z} &\equiv \frac{\overline{\gamma}}{C^{ss}}\,Q^{ss}\Big[\sum_s(\Gamma_s^x)^*\Theta_t^{s,z} + (\Gamma_Q^x)^*\mu_t^{Q,z}\\
&\qquad + (\Theta^{k,x})^*\mathbf{diag}(\mathcal{C}_{kQ})q_t^z + (\mu^{Q,x})^*\mathbf{diag}(\mathcal{C}_{QQ})q_t^z\Big]^*,\\
\mathrm{l}_t^{zz} &\equiv \frac{2\overline{\gamma}}{C^{ss}}\Big[(q_t^z)^*\mathbf{diag}(\mathcal{C}_{kQ})\Theta_t^{k,z} + (q_t^z)^*\mathbf{diag}(\mathcal{C}_{QQ})\mu_t^{Q,z}\Big]Q^{ss},\\
\mathrm{l}_t^{zT} &\equiv \frac{\overline{\gamma}}{C^{ss}}\Big[(q_t^z)^*\mathbf{diag}(\mathcal{C}_{kQ})\Theta_t^{k,T} + (q_t^z)^*\mathbf{diag}(\mathcal{C}_{QQ})\mu_t^{Q,T}\\
&\qquad + (q_t^T)^*\mathbf{diag}(\mathcal{C}_{kQ})\Theta_t^{k,z} + (q_t^T)^*\mathbf{diag}(\mathcal{C}_{QQ})\mu_t^{Q,z}\Big]Q^{ss},
\end{aligned} \tag{C46}$$

where l_t^{TT} follows from the T substitution.

Combining the three contributions in (C38), the interest rate adjustment becomes:

$$\mathcal{R}_t = \epsilon\,\mathcal{R}_{1t} - \frac{\epsilon^2}{2}\Big[\mathrm{D}_{rV}\,v_{2t} + \mathrm{D}_{rQ}\,q_{2t} + \mathrm{D}_{r\mu}\,(\mathcal{L}_t Q)_2 + \mathcal{S}^f_{2t}\Big],$$

where $\mathcal{S}^f_{2t}$ collects the known second-order terms:

$$\mathcal{S}^f_{2t} \equiv \frac{\overline{\gamma}}{C^{ss}}\Big[2\mathcal{S}_{1t}q_{1t} - \frac{2C_{1t}}{C^{ss}}\mathcal{S}_{1t}Q^{ss}\Big] + \mathcal{R}^{0\times 2}_{2t} + \frac{\overline{\gamma}}{C^{ss}}\,\mathcal{S}^{1\times 1}_{2t}\,Q^{ss}. \tag{C47}$$

We express these terms in the blocks of (17):

$$\mathcal{S}^f_{2t} = \sum_{x,y}\overline{\mathbb{J}}^{x,y}\times_3 y\times_2 x + 2\sum_x \bar{\mathrm{J}}^{x,z}_t\,x\,z + 2\sum_x \bar{\mathrm{J}}^{x,T}_t\,x + \bar{\mathrm{j}}^{zz}_t\,z^2 + 2\,\bar{\mathrm{j}}^{zT}_t\,z + \bar{\mathrm{j}}^{TT}_t. \tag{C48}$$

The coefficients combine the three families derived above:

$$\begin{aligned}
&\overline{\mathbb{J}}^{x,y} \equiv \mathbb{J}^{x,y} + \mathbb{K}^{x,y} + \mathbb{L}^{x,y}, \qquad \bar{\mathrm{J}}^{x,z}_t \equiv \mathrm{J}^{x,z}_t + \mathrm{K}^{x,z}_t + \mathrm{L}^{x,z}_t, \qquad \bar{\mathrm{J}}^{x,T}_t \equiv \mathrm{J}^{x,T}_t + \mathrm{K}^{x,T}_t + \mathrm{L}^{x,T}_t,\\
&\bar{\mathrm{j}}^{zz}_t \equiv \mathrm{j}^{zz}_t + \mathrm{k}^{zz}_t + \mathrm{l}^{zz}_t, \qquad \bar{\mathrm{j}}^{zT}_t \equiv \mathrm{j}^{zT}_t + \mathrm{k}^{zT}_t + \mathrm{l}^{zT}_t, \qquad \bar{\mathrm{j}}^{TT}_t \equiv \mathrm{j}^{TT}_t + \mathrm{k}^{TT}_t + \mathrm{l}^{TT}_t.
\end{aligned} \tag{C49}$$

The unknown terms enter the capitalist SAME equations in two ways. First, $\mathrm{D}_{rV}v_{2t}$ and $\mathrm{D}_{rQ}q_{2t}$ contribute to each block its worker and capitalist coefficients, left-multiplied by $-\mathrm{D}_{rV}$ and $-\mathrm{D}_{rQ}$. Second, $\mathrm{D}_{r\mu}(\mathcal{L}_tQ)_2$ combines with the continuation value $\mathcal{L}_tQ$ to give $(\mathbf{I}-\mathrm{D}_{r\mu})(\mathcal{L}_tQ)_2$, recovering the mass matrix of (C21). Finally, the known coefficients in (C49) enter the corresponding SAME forcings with a minus sign. In particular, $\bar{\mathrm{j}}^{TT}_t$ enters the TT block rather than the constant block.

Itô correction (diversified model only). The correction in (C12) is $\iota^Q_i = \mathrm{Y}^Q_i - \frac{\overline{\gamma}}{C}\iota^C_t Q_i + \frac{\overline{\gamma}(\overline{\gamma}+1)}{2}Q_i\|\sigma^C_t\|^2$. Since each term is of order ϵ^2, we use steady-state values and the first-order loadings:

$$\begin{aligned}
\partial_z Q_i &= \epsilon q^z_{it}, & \partial_z \mathcal{C} &= \epsilon \mathcal{C}_Q q^z_t,\\
\partial_{\chi_\ell} Q_i &= Q^\chi_{i\ell}, & \partial_{\chi_\ell}\mathcal{C} &= \overline{\mathcal{C}}^\chi_\ell,\\
\partial_{\mathcal{K}_\ell} Q_i &= Q^k_{i\ell}, & \partial_{\mathcal{K}_\ell}\mathcal{C} &= \overline{\mathcal{C}}^k_\ell,
\end{aligned}$$

where $\overline{\mathcal{C}}^x \equiv \mathcal{C}_x + \mathcal{C}_Q Q^x$ collects the total consumption derivatives. Local-risk terms are proportional to $\sigma_t z + \varsigma_t$, while aggregate risk enters through q^z_t.

We first consider the covariance correction Y^Q_i in (C9). The hedge $\sum_\ell \Sigma_{\Delta,i\ell t}\partial_{\mathcal{K}_\ell}Q_i$ contributes

$\frac{\epsilon^2}{2}\mathrm{d}_{Q\Sigma K}(\sigma_t z+\varsigma_t)$, where:

$$\mathrm{d}_{Q\Sigma K}\equiv 2\,\mathbf{vec}\big((Q^k\Sigma_\Delta)_{ii}\big).$$

Here, we use $\sum_\ell Q^k_{i\ell}A_{i\ell}=(Q^kA)_{ii}$ for symmetric A. The remaining covariance coefficients are:

$$\begin{aligned}\mathrm{D}_{Q\Sigma\chi}&\equiv-\frac{2\overline{\gamma}}{C^{ss}}\,Q^\chi\,\Sigma_\chi\,\mathbf{diag}(\overline{\mathcal{C}}^\chi),\qquad \mathrm{D}_{Q\Sigma\Delta}\equiv-\frac{2\overline{\gamma}}{C^{ss}}\,Q^k\,\Sigma_\Delta\,\mathbf{diag}(\overline{\mathcal{C}}^k),\\ \mathrm{d}_{Q\Sigma C}&\equiv-\frac{2\overline{\gamma}}{C^{ss}}\,\mathbf{diag}(Q_i^{ss})\,\Sigma_\Delta\,(\overline{\mathcal{C}}^k)^*,\qquad \mathrm{d}_{Q\Sigma z,t}\equiv-\frac{2\overline{\gamma}}{C^{ss}}\,(\mathcal{C}_Q q_t^z)\,q_t^z.\end{aligned}\tag{C50}$$

The first two matrices and $\mathrm{d}_{Q\Sigma z,t}$ capture the price channel. The term $\mathrm{d}_{Q\Sigma C}$ captures the covariance between the stochastic discount factor and changes in the local capital stock.

We next combine the consumption Itô correction ι_t^C in (C5) with the log-capital drift correction $-\Sigma^2_{\Delta,it}/2$. Using the level-capital Hessian $K_i^2\mathcal{C}_{i,KK}=\mathcal{C}_{i,\mathcal{K}\mathcal{K}}-\mathcal{C}_{i,\mathcal{K}}=-\phi(1-\phi)R_i^{ss}K_i^{ss}$, we obtain, to second order:

$$\iota_t^C-\frac{1}{2}\sum_\ell\mathcal{C}_{\ell,\mathcal{K}}\,\Sigma^2_{\Delta,\ell t}=\frac{\epsilon^2}{2}\Big[\bar{\mathrm{r}}_t+\big(\mathrm{r}^{\Sigma\chi}+\mathrm{r}^{\Sigma\Delta}\big)(\sigma_t z+\varsigma_t)\Big],\tag{C51}$$

where:

$$\begin{aligned}\mathrm{r}^{\Sigma\chi}&\equiv\Big[\mathcal{C}^*_{\chi\chi}\,\Sigma^2_\chi+\mathcal{C}^*_{QQ}\,\mathbf{diag}\big(Q^\chi\Sigma_\chi(Q^\chi)^*\big)\Big]\mathbb{1},\qquad \bar{\mathrm{r}}_t\equiv\mathcal{C}^*_{QQ}\big(q_t^z\cdot q_t^z\big),\\ \mathrm{r}^{\Sigma\Delta}&\equiv\Big[-\phi(1-\phi)\,\mathcal{C}^*_{\chi\chi}\,\Sigma^2_\Delta+\mathcal{C}^*_{QQ}\,\mathbf{diag}\big(Q^k\Sigma_\Delta(Q^k)^*\big)+2\,\mathcal{C}^*_{kQ}\,\mathbf{diag}\big(Q^k\Sigma_\Delta\big)\Big]\mathbb{1}.\end{aligned}\tag{C52}$$

Finally, the precautionary term depends on the variance of consumption growth. Define:

$$\mathrm{s}^{\Sigma\chi}\equiv\frac{1}{(C^{ss})^2}\,\overline{\mathcal{C}}^\chi\,\Sigma_\chi\,(\overline{\mathcal{C}}^\chi)^*,\qquad \mathrm{s}^{\Sigma\Delta}\equiv\frac{1}{(C^{ss})^2}\,\overline{\mathcal{C}}^k\,\Sigma_\Delta\,(\overline{\mathcal{C}}^k)^*,\qquad \bar{\mathrm{s}}_t\equiv\frac{\big(\mathcal{C}_Q q_t^z\big)^2}{(C^{ss})^2},\tag{C53}$$

so that, to second order, $\|\sigma_t^C\|^2=\epsilon^2[\bar{\mathrm{s}}_t+(\mathrm{s}^{\Sigma\chi}+\mathrm{s}^{\Sigma\Delta})(\sigma_t z+\varsigma_t)]$. Combining the three contributions gives, to second order:

$$\begin{aligned}\iota_t^Q=\frac{\epsilon^2}{2}\Big\{&\mathrm{d}_{Q\Sigma z,t}+\Big[\mathrm{D}_{Q\Sigma\chi}\mathbb{1}+\mathrm{D}_{Q\Sigma\Delta}\mathbb{1}+\mathrm{d}_{Q\Sigma K}+\mathrm{d}_{Q\Sigma C}\Big](\sigma_t z+\varsigma_t)\\ &-\frac{\overline{\gamma}}{C^{ss}}\Big[\bar{\mathrm{r}}_t+\big(\mathrm{r}^{\Sigma\chi}+\mathrm{r}^{\Sigma\Delta}\big)(\sigma_t z+\varsigma_t)\Big]Q^{ss}\\ &+\overline{\gamma}(\overline{\gamma}+1)\Big[\bar{\mathrm{s}}_t+\big(\mathrm{s}^{\Sigma\chi}+\mathrm{s}^{\Sigma\Delta}\big)(\sigma_t z+\varsigma_t)\Big]Q^{ss}\Big\}.\end{aligned}\tag{C54}$$

All coefficients depend only on parameters, steady-state values, and the FAME solution.

C.4.2 SAME in Vector Notation

We now organize the second-order approximations into the blocks of Proposition 5 and the law of motion. Identifying coefficients in (16) and (17) gives one equation for each unknown. As in the main text, we stack the worker equations above the capitalist equations. Each block is linear, and the solution is recursive. The deterministic SAME depends only on the steady state and the FAME solution. The cross blocks use the deterministic tensors, the zz, TT, and zT blocks use the cross solutions, the local-risk blocks use the deterministic tensors, and the constant block uses the zz solution. We solve them in this order, so each block has a known forcing. Only the deterministic SAME involves solving for a tensor. The remaining blocks are linear differential equations in time, as in the FAME.

To simplify notation, we use $\widetilde{\mathbf{I}}$ for two matrices of different dimensions. In equations for capitalists alone, $\widetilde{\mathbf{I}}$ denotes the $\mathcal{I} \times \mathcal{I}$ matrix $\mathbf{I} - \mathrm{D}_{r\mu}$, which multiplies the capital-price drift as in the FAME. When we stack the worker and capitalist equations, $\widetilde{\mathbf{I}}$ denotes the $2\mathcal{I} \times 2\mathcal{I}$ mass matrix of (C21), with the identity on the worker rows and $\mathbf{I} - \mathrm{D}_{r\mu}$ on the capitalist rows. The dimensions of the object it multiplies distinguish the two cases.

Deterministic SAME. Collecting the coefficients of each pair (x, y) from Appendix C.4, we obtain:

$$\begin{aligned}
\rho\, \mathcal{V}^{x,y} &= \mathbb{V}_0^{x,y} + \mathcal{V}^{x,y} \times_1 (M + \mathrm{X}_V) + \mathcal{Q}^{x,y} \times_1 A_V + \sum_s \Big[\mathcal{V}^{s,y} \times_2 \Theta^{s,x} + \mathcal{V}^{x,s} \times_3 \Theta^{s,y} \Big], \\
\rho\, \mathcal{Q}^{x,y} &= \mathbb{Q}_0^{x,y} + \mathcal{V}^{x,y} \times_1 \big(\widetilde{\mathbf{I}} \mathrm{X}_Q - \mathrm{D}_{rV} \big) + \mathcal{Q}^{x,y} \times_1 \big(\widetilde{\mathbf{I}} A_Q - \mathrm{D}_{rQ} \big) \\
&\quad + \sum_s \Big[\mathcal{Q}^{s,y} \times_2 \Theta^{s,x} + \mathcal{Q}^{x,s} \times_3 \Theta^{s,y} \Big] \times_1 \widetilde{\mathbf{I}},
\end{aligned}$$

where the forcing tensors are:

$$\begin{aligned}
\mathbb{V}_0^{x,y} &\equiv \mathbb{D}_{Uxy} + \mu\nu \big(\mathbb{V}^{x,y} \times_1 m + \mathbb{M} \times_3 V^y \times_2 V^x \big) + \mathbb{1}_{x=n} \mathbb{Y}_V \times_3 V^y + \mathbb{1}_{y=n} \mathbb{Y}_V^* \times_2 V^x \\
&\quad + \mathbb{Z}_V \times_3 V^y \times_2 V^x + \mathbb{Q}^{x,y} \times_1 B_V, \\
\mathbb{Q}_0^{x,y} &\equiv \mathbb{D}_{Rxy} + \mathbb{Q}^{x,y} \times_1 \mathrm{D}_{\mathcal{I}Q} - \overline{\mathbb{J}}^{x,y} \\
&\quad + \Big[\mathbb{1}_{x=n} \mathbb{Y}_Q \times_3 V^y + \mathbb{1}_{y=n} \mathbb{Y}_Q^* \times_2 V^x + \mathbb{Z}_Q \times_3 V^y \times_2 V^x + \mathbb{Q}^{x,y} \times_1 B_Q \Big] \times_1 \widetilde{\mathbf{I}}.
\end{aligned} \tag{C55}$$

Productivity mean reversion is already captured by $\Theta^{\chi,\chi} = -\kappa\mathbf{I}$ in the sums over s. Thus, pairs with one χ index have a contribution of $-\kappa$, while the pair (χ,χ) has a contribution of -2κ. As a result, we do not need a separate productivity block.

We now stack the worker and capitalist tensors as:

$$\mathcal{T} \equiv \left[\mathcal{V}^{x,y}\right]_{x,y} \odot \left[\mathcal{Q}^{x,y}\right]_{x,y}, \qquad \mathbb{T}_0 \equiv \left[\mathbb{V}_0^{x,y}\right]_{x,y} \odot \left[\mathbb{Q}_0^{x,y}\right]_{x,y},$$

where $[\,\cdot\,]_{x,y}$ collects the 3×3 array of pairs, and $\odot$ stacks the worker and capitalist arrays into the front and back panels of a $2\mathcal{I}\times 3\mathcal{I}\times 3\mathcal{I}$ tensor. Defining the matrix Φ, the deterministic SAME becomes:

$$\Phi \equiv \begin{pmatrix} M + \mathrm{X}_V & A_V \\ \widetilde{\mathbf{I}}\mathrm{X}_Q - \mathrm{D}_{rV} & \widetilde{\mathbf{I}}A_Q - \mathrm{D}_{rQ} \end{pmatrix},$$
$$\rho\,\mathcal{T} = \mathbb{T}_0 + \mathcal{T}\times_1\Phi + \Big[\mathcal{T}\times_2\Theta + \mathcal{T}\times_3\Theta\Big]\times_1\widetilde{\mathbf{I}}, \tag{C56}$$

where $\widetilde{\mathbf{I}}$ acts on the panel index and Θ is the $3\mathcal{I}\times 3\mathcal{I}$ transition matrix of Appendix C.3.2. This recovers the first equation of Proposition 5.

Using the inverse (C23) and defining $\overline{\Phi} \equiv \widetilde{\mathbf{I}}^{-1}\Phi$, we obtain the integral representation:

$$\mathcal{T} = \int_0^\infty \left(\mathbb{T}_0\times_1\widetilde{\mathbf{I}}^{-1}\right)\times_1 e^{(\overline{\Phi}-\rho\widetilde{\mathbf{I}}^{-1})t}\times_2 e^{\Theta t}\times_3 e^{\Theta t}\,dt,$$

which we evaluate using the eigendecompositions of $\overline{\Phi}-\rho\widetilde{\mathbf{I}}^{-1}$ and Θ. These decompositions turn each matrix exponential into a diagonal factor acting on one index of $\mathbb{T}_0\times_1\widetilde{\mathbf{I}}^{-1}$, so we can compute the integral over t with a few quadrature nodes. Thus, instead of solving a linear system for the $18\mathcal{I}^3$ entries of $\mathcal{T}$, we compute two eigendecompositions of dimensions $2\mathcal{I}$ and $3\mathcal{I}$. We use the same decompositions for the cross and stochastic blocks below.

Cross SAME. Collecting the coefficients of xz for each x gives:

$$(\rho+\theta)V_t^{x,z} - \frac{\partial V_t^{x,z}}{\partial t} = \mathbf{V}_{0t}^{x,z} + (M+\mathrm{X}_V)V_t^{x,z} + A_V Q_t^{x,z} + \sum_s V_t^{s,z}\Theta^{s,x},$$
$$(\rho\mathbf{I}+\theta\widetilde{\mathbf{I}})Q_t^{x,z} - \widetilde{\mathbf{I}}\frac{\partial Q_t^{x,z}}{\partial t} = \mathbf{Q}_{0t}^{x,z} + \left(\widetilde{\mathbf{I}}\mathrm{X}_Q - \mathrm{D}_{rV}\right)V_t^{x,z}$$
$$+ \left(\widetilde{\mathbf{I}}A_Q - \mathrm{D}_{rQ}\right)Q_t^{x,z} + \widetilde{\mathbf{I}}\sum_s Q_t^{s,z}\Theta^{s,x},$$

where the forcing terms are:

$$\begin{aligned}
\mathbf{V}_{0t}^{x,z} &\equiv \mu\nu\big(m\mathbf{diag}(v_t^z)V^x + \mathbb{M}\times_2 V^x \times_3 v_t^z\big) + \mathbb{1}_{x=n}\mathbb{Y}_V \times_3 v_t^z + \mathbb{Z}_V \times_2 V^x \times_3 v_t^z \\
&\quad + B_V\mathbf{diag}(q_t^z)Q^x + \tfrac{1}{2}\sum_s \Big[\mathcal{V}^{s,x}\times_2 \Theta_t^{s,z} + \mathcal{V}^{x,s}\times_3 \Theta_t^{s,z}\Big], \\
\mathbf{Q}_{0t}^{x,z} &\equiv \mathrm{D}_{\mathcal{I}Q}\mathbf{diag}(q_t^z)Q^x + \mathrm{D}_{QQT}Q^x\sigma_t - \bar{\mathrm{j}}_t^{x,z} \\
&\quad + \widetilde{\mathbf{I}}\Big[\mathbb{1}_{x=n}\mathbb{Y}_Q\times_3 v_t^z + \mathbb{Z}_Q\times_2 V^x\times_3 v_t^z + B_Q\mathbf{diag}(q_t^z)Q^x \\
&\qquad + \tfrac{1}{2}\sum_s\Big(\mathcal{Q}^{s,x}\times_2\Theta_t^{s,z} + \mathcal{Q}^{x,s}\times_3\Theta_t^{s,z}\Big)\Big].
\end{aligned}$$

The corresponding trend expressions follow from the T substitution.

We now stack the six cross matrices into $\mathbf{V}_t^z \equiv (V_t^{x,z}; Q_t^{x,z})_x$ and their forcings into $\mathbf{V}_{0t}^z \equiv (\mathbf{V}_{0t}^{x,z}; \mathbf{Q}_{0t}^{x,z})_x$, both of dimension $2\mathcal{I}\times 3\mathcal{I}$. The sums over s correspond to right multiplication by Θ, giving:

$$\begin{aligned}
(\rho\mathbf{I} + \theta\widetilde{\mathbf{I}})\,\mathbf{V}_t^z - \widetilde{\mathbf{I}}\frac{\partial\mathbf{V}_t^z}{\partial t} &= \mathbf{V}_{0t}^z + \Phi\,\mathbf{V}_t^z + \widetilde{\mathbf{I}}\,\mathbf{V}_t^z\,\Theta, \\
\rho\,\mathbf{V}_t^T - \widetilde{\mathbf{I}}\frac{\partial\mathbf{V}_t^T}{\partial t} &= \mathbf{V}_{0t}^T + \Phi\,\mathbf{V}_t^T + \widetilde{\mathbf{I}}\,\mathbf{V}_t^T\,\Theta,
\end{aligned} \tag{C57}$$

which are the two Sylvester differential equations of Proposition 5.

Stochastic SAME. Collecting the z^2 and z terms, the time-dependent constant terms, and the local-risk terms $\sigma_t z$ and ς_t gives five pairs of vector equations with the same operator. For $\mathbf{v}_t \in \{\mathbf{v}_t^{z,z}, \mathbf{v}_t^{T,T}, \mathbf{v}_t^{z,T}, \mathbf{v}_t^{\Sigma,z}, \mathbf{v}_t^{\Sigma,T}\}$, with corresponding shifts $\theta_{\mathbf{v}} \in \{2\theta, 0, \theta, \theta, 0\}$, we obtain:

$$(\rho\mathbf{I} + \theta_{\mathbf{v}}\widetilde{\mathbf{I}})\,\mathbf{v}_t - \widetilde{\mathbf{I}}\frac{\partial\mathbf{v}_t}{\partial t} = \mathbf{v}_{0t} + \Phi\,\mathbf{v}_t, \tag{C58}$$

which gives the vector equations of Proposition 5. The forcing terms for the zz block are:

$$\begin{aligned}
\mathbf{v}_{0t}^{z,z} &\equiv \mu\nu\big(m(v_t^z\cdot v_t^z) + \mathbb{M}\times_3 v_t^z\times_2 v_t^z\big) + \mathbb{Z}_V\times_3 v_t^z\times_2 v_t^z + B_V(q_t^z\cdot q_t^z) + 2\sum_s V_t^{s,z}\Theta_t^{s,z}, \\
\mathbf{q}_{0t}^{z,z} &\equiv \mathrm{D}_{\mathcal{I}Q}(q_t^z\cdot q_t^z) + 2\mathrm{D}_{QQT}q_t^z\sigma_t - \mathrm{j}_t^{zz} \\
&\quad + \widetilde{\mathbf{I}}\Big[\mathbb{Z}_Q\times_3 v_t^z\times_2 v_t^z + B_Q(q_t^z\cdot q_t^z) + 2\sum_s Q_t^{s,z}\Theta_t^{s,z}\Big],
\end{aligned}$$

with the TT forcings obtained through the T substitution. The zT block combines the stochastic and trend responses:

$$\begin{aligned}
\mathbf{v}_{0t}^{z,T} &\equiv \mu\nu\big(m(v_t^z \cdot v_t^T) + \mathbb{M} \times_3 v_t^z \times_2 v_t^T\big) + \mathbb{Z}_V \times_3 v_t^z \times_2 v_t^T + B_V(q_t^z \cdot q_t^T) \\
&\quad + \sum_s \Big[V_t^{s,z}\Theta_t^{s,T} + V_t^{s,T}\Theta_t^{s,z}\Big], \\
\mathbf{q}_{0t}^{z,T} &\equiv \mathrm{D}_{\mathcal{I}Q}(q_t^z \cdot q_t^T) + \mathrm{D}_{QQT}\big(q_t^z T_t + q_t^T \sigma_t\big) - \bar{\mathrm{j}}_t^{zT} \\
&\quad + \widetilde{\mathbf{I}}\Big[\mathbb{Z}_Q \times_3 v_t^z \times_2 v_t^T + B_Q(q_t^z \cdot q_t^T) + \sum_s \Big(Q_t^{s,z}\Theta_t^{s,T} + Q_t^{s,T}\Theta_t^{s,z}\Big)\Big].
\end{aligned}$$

Finally, the local-risk forcings are:

$$\begin{aligned}
\mathbf{v}_{0t}^{\Sigma,z} &\equiv \Big[V^k \Sigma_\Delta^2 \mathrm{d}_{\mathcal{I}\Sigma T} + \mathcal{V}^{k,k} \times_{2,3} \Sigma_\Delta + \mathcal{V}^{\chi,\chi} \times_{2,3} \Sigma_\chi\Big]\sigma_t, \\
\mathbf{q}_{0t}^{\Sigma,z} &\equiv \Big[\mathrm{d}_{Q\Sigma K} + \mathrm{d}_{Q\Sigma C} + \big(\mathrm{D}_{Q\Sigma\chi} + \mathrm{D}_{Q\Sigma\Delta}\big)\mathbb{1} \\
&\qquad - \frac{\overline{\gamma}}{C^{ss}}\big(\mathrm{r}^{\Sigma\chi} + \mathrm{r}^{\Sigma\Delta}\big)Q^{ss} + \overline{\gamma}(\overline{\gamma}+1)\big(\mathrm{s}^{\Sigma\chi} + \mathrm{s}^{\Sigma\Delta}\big)Q^{ss}\Big]\sigma_t \\
&\qquad + \widetilde{\mathbf{I}}\Big[Q^k \Sigma_\Delta^2 \mathrm{d}_{\mathcal{I}\Sigma T} + \mathcal{Q}^{k,k} \times_{2,3} \Sigma_\Delta + \mathcal{Q}^{\chi,\chi} \times_{2,3} \Sigma_\chi\Big]\sigma_t,
\end{aligned}$$

where $\mathbf{v}_{0t}^{\Sigma,T}$ and $\mathbf{q}_{0t}^{\Sigma,T}$ follow by replacing σ_t with ς_t.

Constant SAME. The remaining constant terms determine the ex-ante valuation of risk. They satisfy the vector equation of Proposition 5, with $\theta_{\mathbf{v}} = 0$:

$$\begin{aligned}
\rho\,\overline{\mathbf{v}}_t - \widetilde{\mathbf{I}}\frac{\partial \overline{\mathbf{v}}_t}{\partial t} &= \overline{\mathbf{v}}_{0t} + \Phi\,\overline{\mathbf{v}}_t, \qquad \overline{\mathbf{v}}_{0t} \equiv v_t^{z,z}, \\
\overline{\mathbf{q}}_{0t} &\equiv \mathrm{d}_{Q\Sigma z,t} - \frac{\overline{\gamma}}{C^{ss}}\overline{\mathrm{r}}_t Q^{ss} + \overline{\gamma}(\overline{\gamma}+1)\overline{\mathrm{s}}_t Q^{ss} + \widetilde{\mathbf{I}}\, q_t^{z,z}.
\end{aligned}$$

Second-order laws of motion. The drifts (C35) and (C36) give the law of motion of the state deviations to second order:

$$\begin{aligned}
\frac{dn_t}{dt} &= M^* n_t + G v_{1t} + \frac{\epsilon}{2}\Big\{G v_{2t} + 2\,\mathbb{G} \times_3 v_{1t} \times_2 n_t + (\mathbb{H} \times_2 N) \times_3 v_{1t} \times_2 v_{1t}\Big\}, \\
\frac{dk_t}{dt} &= \mathrm{D}_{\mathcal{I}Q} q_{1t} + \mathrm{d}_{\mathcal{I}T}\widehat{T}_t + \frac{\epsilon}{2}\Big\{\mathrm{D}_{\mathcal{I}Q} q_{2t} + \mathrm{D}_{\mathcal{I}QQ}(q_{1t} \cdot q_{1t}) + \Sigma_\Delta^2 \mathrm{d}_{\mathcal{I}\Sigma T}(\sigma_t z + \varsigma_t)\Big\}, \\
\frac{d\widehat{\chi}_t}{dt} &= -\kappa\widehat{\chi}_t + \mathrm{d}_{XT}\widehat{T}_t.
\end{aligned} \tag{C59}$$

The productivity drift has no second-order term because it is linear in $\widehat{\chi}_t$.

Island model. The island economy differs from the diversified economy in three ways. First, there is no risk-free-rate adjustment, so the two diversified-only derivations in Appendix C.4 do not apply. We therefore set $\widetilde{\mathbf{I}} = \mathbf{I}$, while the matrices D_{rV}, D_{rQ}, $\mathrm{D}_{r\mu}$, the families $\overline{\mathbb{J}}$, $\bar{\mathrm{J}}$, $\bar{\mathrm{j}}$, and the rebate and precautionary scalars are absent. Second, the capitalists' flow payoff changes. Third, the investment rule depends on the rental rate, affecting the capital drift and its contribution to continuation values. The worker flow, migration, and population terms remain unchanged.

We first consider the capitalists' flow payoff, given by the right-hand side of (C14) excluding the continuation term:

$$W_{it} \equiv \Omega_i(Q_{it}, R_{it}) - \Big[\Delta_{it} + \frac{\overline{\gamma}}{2}\Sigma^2_{\Delta,it}\Big] Q_{it} + (1-\overline{\gamma}) \sum_{\ell} \Sigma_{\Delta,i\ell t} \frac{\partial Q_{it}}{\partial \mathcal{K}_\ell} - \frac{\overline{\gamma}}{2Q_{it}} \|\sigma^Q_{it}\|^2.$$

Holding Q fixed, the other state variables affect Ω_i through the rental rate. Its second derivatives at steady state are:

$$\begin{aligned} \Omega^{ss}_{i,QQ} &= -\frac{\rho}{Q^{ss}_i} \frac{\overline{\gamma}\rho + (2\overline{\gamma}-1)\zeta\Delta_i}{\rho + \overline{\gamma}\zeta\Delta_i}, \qquad \Omega^{ss}_{i,QR} = \frac{\overline{\gamma}}{c_i'(\Delta_i)} \frac{\rho + \zeta\Delta_i}{\rho + \overline{\gamma}\zeta\Delta_i}, \\ \Omega^{ss}_{i,RR} &= -\frac{\overline{\gamma}\, Q^{ss}_i}{c_i'(\Delta_i)^2(\rho + \overline{\gamma}\zeta\Delta_i)}. \end{aligned} \tag{C60}$$

To apply Lemma 4, define $\overline{\Omega}_i \equiv \Omega^{ss}_{i,R} R^{ss}_i + \Omega^{ss}_{i,RR}(R^{ss}_i)^2$, with $\Omega^{ss}_{i,R}$ from (C30). The lemma then gives:

$$\begin{aligned} \mathbb{D}_{Qxy} &\equiv \mathbf{diag}_3\big(\overline{\Omega}_i\, \mathrm{e}^R_{x,i}\mathrm{e}^R_{y,i}\big) - \mathbb{1}_{x=y=n}\, \mathbf{diag}_3\Big(\frac{\psi \Omega^{ss}_{i,R} R^{ss}_i}{(N^{ss}_i)^2}\Big), \qquad \mathrm{D}_{QQQ} \equiv \mathbf{diag}\big(\Omega^{ss}_{i,QQ}\big), \\ \mathbb{D}_{QQx} &\equiv \mathbf{diag}_3\big(\Omega^{ss}_{i,QR} R^{ss}_i\, \mathrm{e}^R_{x,i}\big), \qquad \mathrm{D}_{QQT} \equiv -\mathbf{diag}(\delta_i). \end{aligned}$$

The depreciation, hedge, and valuation-variance terms give the following coefficients:

$$\begin{aligned} \mathrm{d}_{Q\Sigma T} &\equiv -\overline{\gamma}\, \mathbf{vec}\big(Q^{ss}_i\big), \qquad \mathrm{d}_{Q\Sigma K} \equiv 2(1-\overline{\gamma})\, \mathbf{vec}\big((Q^k \Sigma_\Delta)_{ii}\big), \\ \mathrm{d}_{Q\Sigma Q} &\equiv -\overline{\gamma}\, \mathbf{vec}\Big(\frac{(Q^k\Sigma_\Delta(Q^k)^*)_{ii} + (Q^\chi \Sigma_\chi (Q^\chi)^*)_{ii}}{Q^{ss}_i}\Big), \qquad \mathrm{d}_{Q\Sigma z,t} \equiv -\overline{\gamma}\, \frac{q^z_t \cdot q^z_t}{Q^{ss}}. \end{aligned}$$

Using the first-order matrices from the island FAME and collecting terms as in (17), the flow

expands as:

$$
\begin{aligned}
W_t - W^{ss} = \epsilon\Big\{ & \sum_x \mathrm{D}_{Qx} x + \mathrm{d}_{QT}\widehat{T}_t \Big\} \\
& + \frac{\epsilon^2}{2}\Big\{ \sum_{x,y} \left[\mathbb{D}_{Qxy} + \mathbb{Q}^{x,y} \times_1 \mathrm{D}_{QQQ} + \mathbb{D}_{QQx} \times_3 Q^y + \mathbb{D}_{QQy} \times_2 Q^x\right] \times_3 y \times_2 x \\
& \quad + 2\sum_x \left[\mathrm{D}_{QQQ}\mathbf{diag}(q_t^z)Q^x + \mathrm{D}_{QQT}Q^x\sigma_t + \mathbf{diag}\big(\Omega^{ss}_{i,QR}R_i^{ss}\mathrm{e}^R_{x,i}q_{it}^z\big)\right] x\, z \\
& \quad + 2\sum_x \left[\mathrm{D}_{QQQ}\mathbf{diag}(q_t^T)Q^x + \mathrm{D}_{QQT}Q^x T_t + \mathbf{diag}\big(\Omega^{ss}_{i,QR}R_i^{ss}\mathrm{e}^R_{x,i}q_{it}^T\big)\right] x \\
& \quad + \left[\mathrm{D}_{QQQ}(q_t^z \cdot q_t^z) + 2\,\mathrm{D}_{QQT}q_t^z\sigma_t\right]z^2 + \left[\mathrm{D}_{QQQ}(q_t^T \cdot q_t^T) + 2\,\mathrm{D}_{QQT}q_t^T T_t\right] \\
& \quad + 2\left[\mathrm{D}_{QQQ}(q_t^z \cdot q_t^T) + \mathrm{D}_{QQT}(q_t^z T_t + q_t^T\sigma_t)\right]z \\
& \quad + \left[\Sigma_\Delta^2 \mathrm{d}_{Q\Sigma T} + \mathrm{d}_{Q\Sigma K} + \mathrm{d}_{Q\Sigma Q}\right](\sigma_t z + \varsigma_t) + \mathrm{d}_{Q\Sigma z,t}\Big\}.
\end{aligned}
$$

We next consider the continuation value from changes in the capital distribution. Unlike in the diversified economy, investment $g_i(Q,R)$ responds directly to the rental rate. Let $\widehat{R}_{1t} \equiv \sum_x \mathrm{D}_{Rx}x$. The first- and second-order drifts are:

$$
\begin{aligned}
\mu^k_{1t} &= \sum_x \left(\mathrm{D}_{\mathcal{I}Q}Q^x + \mathrm{D}_{\mathcal{I}x}\right)x + \left(\mathrm{D}_{\mathcal{I}Q}q_t^z + \mathrm{d}_{\mathcal{I}T}\sigma_t\right)z + \left(\mathrm{D}_{\mathcal{I}Q}q_t^T + \mathrm{d}_{\mathcal{I}T}T_t\right), \\
\mu^k_{2t} &= \tfrac{1}{2}\Big[\sum_{x,y}\mathbb{D}_{\mathcal{I}xy} \times_3 y \times_2 x + \mathrm{D}_{\mathcal{I}Q}q_{2t} + \mathrm{D}_{\mathcal{I}QQ}(q_{1t}\cdot q_{1t}) \\
&\qquad + 2\,\mathrm{D}_{\mathcal{I}QR}(q_{1t}\cdot \widehat{R}_{1t}) + \Sigma_\Delta^2 \mathrm{d}_{\mathcal{I}\Sigma T}(\sigma_t z + \varsigma_t)\Big].
\end{aligned}
$$

The first-order matrices $\mathrm{D}_{\mathcal{I}Q}$ and $\mathrm{D}_{\mathcal{I}x}$ are defined in (C33). Differentiating the investment rule twice gives:

$$
\begin{aligned}
\mathcal{G}^{ss}_{i,QQ} &= -\frac{(1-\overline{\gamma})\,\rho\zeta\Delta_i\left[(1+(\overline{\gamma}-1)\zeta)\rho^2 + \overline{\gamma}(3-\overline{\gamma})\rho\zeta\Delta_i + \overline{\gamma}\zeta^2\Delta_i^2\right]}{(Q_i^{ss})^2(\rho+\overline{\gamma}\zeta\Delta_i)^3}, \\
\mathcal{G}^{ss}_{i,QR} &= \frac{(1-\overline{\gamma})\,\overline{\gamma}\rho\zeta^2\Delta_i\left[\rho-(\overline{\gamma}-1)\Delta_i\right]}{c_i'(\Delta_i)\,Q_i^{ss}(\rho+\overline{\gamma}\zeta\Delta_i)^3}, \qquad \mathcal{G}^{ss}_{i,RR} = \frac{\overline{\gamma}\zeta\Delta_i\left[(\overline{\gamma}\zeta-1)\rho-\overline{\gamma}^2\zeta\Delta_i\right]}{c_i'(\Delta_i)^2(\rho+\overline{\gamma}\zeta\Delta_i)^3}.
\end{aligned} \tag{C61}
$$

Defining $\overline{g}_i \equiv g^{ss}_{i,R} R^{ss}_i + g^{ss}_{i,RR}(R^{ss}_i)^2$, Lemma 4 gives:

$$\mathbb{D}_{\mathcal{I}xy} \equiv \mathbf{diag}_3\big(\overline{g}_i\, \mathrm{e}^R_{x,i}\mathrm{e}^R_{y,i}\big) - \mathbb{1}_{x=y=n}\, \mathbf{diag}_3\Big(\frac{\psi g^{ss}_{i,R} R^{ss}_i}{(N^{ss}_i)^2}\Big),$$

$$\mathrm{D}_{\mathcal{I}QQ} \equiv \mathbf{diag}\big(g^{ss}_{i,QQ}\big), \qquad \mathrm{D}_{\mathcal{I}QR} \equiv \mathbf{diag}\big(g^{ss}_{i,QR}\big), \qquad \mathbb{D}_{\mathcal{I}Qx} \equiv \mathbf{diag}_3\big(g^{ss}_{i,QR} R^{ss}_i\, \mathrm{e}^R_{x,i}\big),$$

$$\mathrm{J}^{x,z}_{\mathcal{I},t} \equiv \mathbf{diag}\big(g^{ss}_{i,QR} R^{ss}_i\, \mathrm{e}^R_{x,i}\, q^z_{it}\big), \qquad \mathbb{J}^{x,y}_{\mathcal{I}} \equiv \mathbb{D}_{\mathcal{I}Qx} \times_3 Q^y + \mathbb{D}_{\mathcal{I}Qy} \times_2 Q^x.$$

As in the diversified economy, the continuation value is $\epsilon W^k \mu^k_{1t} + \frac{\epsilon^2}{2}[2W^k \mu^k_{2t} + (\partial_k w_2)\mu^k_{1t}]$ for $W \in \{V, Q\}$. We apply Lemma 3 using $\Theta^{k,x} = \mathrm{D}_{\mathcal{I}Q} Q^x + \mathrm{D}_{\mathcal{I}x}$. Relative to Appendix C.4, the first-order term gains $W^k \sum_x \mathrm{D}_{\mathcal{I}x} x$, while the second-order terms gain:

$$\sum_{x,y} \big[\mathbb{D}_{\mathcal{I}xy} + \mathbb{J}^{x,y}_{\mathcal{I}}\big] \times_1 W^k \times_3 y \times_2 x + 2\sum_x W^k \mathrm{J}^{x,z}_{\mathcal{I},t}\, x\, z + 2\sum_x W^k \mathrm{J}^{x,T}_{\mathcal{I},t}\, x.$$

We now incorporate these changes into the SAME blocks. The corresponding value operator and capital transition are:

$$\Phi = \begin{pmatrix} M + \mathrm{X}_V & A_V \\ \mathrm{X}_Q & A_Q \end{pmatrix}, \qquad A_W \equiv W^k \mathrm{D}_{\mathcal{I}Q}, \qquad B_W \equiv W^k \mathrm{D}_{\mathcal{I}QQ},$$

$$\Theta^{k,x} = \mathrm{D}_{\mathcal{I}Q} Q^x + \mathrm{D}_{\mathcal{I}x}, \qquad \Theta^{k,z}_t = \mathrm{D}_{\mathcal{I}Q} q^z_t + \mathrm{d}_{\mathcal{I}T}\sigma_t.$$

The worker forcing tensor $\mathbb{V}^{x,y}_0$ gains two terms, $\mathbb{D}_{\mathcal{I}xy} \times_1 V^k$ and $\mathbb{J}^{x,y}_{\mathcal{I}} \times_1 V^k$. The capitalist forcing tensor $\mathbb{Q}^{x,y}_0$ becomes:

$$\begin{aligned}\mathbb{Q}^{x,y}_0 \equiv\ & \mathbb{D}_{Qxy} + \mathbb{Q}^{x,y} \times_1 \mathrm{D}_{QQQ} + \mathbb{D}_{QQx} \times_3 Q^y + \mathbb{D}_{QQy} \times_2 Q^x \\ & + \mathbb{1}_{x=n} \mathbb{Y}_Q \times_3 V^y + \mathbb{1}_{y=n} \mathbb{Y}^*_Q \times_2 V^x + \mathbb{Z}_Q \times_3 V^y \times_2 V^x \\ & + \mathbb{D}_{\mathcal{I}xy} \times_1 Q^k + \mathbb{Q}^{x,y} \times_1 B_Q + \mathbb{J}^{x,y}_{\mathcal{I}} \times_1 Q^k.\end{aligned}$$

The deterministic SAME retains the form of (C56), with $\widetilde{\mathbf{I}} = \mathbf{I}$.

For the cross blocks, both workers and capitalists have the additional term $W^k \mathrm{J}^{x,z}_{\mathcal{I},t}$. In the capitalist forcing, the term $\mathrm{D}_{\mathcal{I}Q}\mathbf{diag}(q^z_t)Q^x$ of $\mathbf{Q}^{x,z}_{0t}$ is replaced by:

$$\mathrm{D}_{QQQ}\mathbf{diag}(q^z_t)Q^x + \mathbf{diag}\big(\Omega^{ss}_{i,QR} R^{ss}_i \mathrm{e}^R_{x,i} q^z_{it}\big).$$

Similarly, the aggregate forcings use $\mathrm{D}_{QQQ}(q^z_t \cdot q^z_t)$ in place of $\mathrm{D}_{\mathcal{I}Q}(q^z_t \cdot q^z_t)$, with corresponding

changes in the TT and zT blocks. The capitalists' Σ and constant forcings are:

$$\begin{aligned}\mathbf{q}_{0t}^{\Sigma,z} &\equiv \Big[\Sigma_\Delta^2 \mathrm{d}_{Q\Sigma T} + \mathrm{d}_{Q\Sigma K} + \mathrm{d}_{Q\Sigma Q} + Q^k \Sigma_\Delta^2 \mathrm{d}_{\mathcal{I}\Sigma T} \\ &\qquad + \mathcal{Q}^{k,k} \times_{2,3} \Sigma_\Delta + \mathcal{Q}^{\chi,\chi} \times_{2,3} \Sigma_\chi\Big]\sigma_t, \\ \overline{\mathbf{q}}_{0t} &\equiv \mathrm{d}_{Q\Sigma z,t} + q_t^{z,z},\end{aligned}$$

where $\mathbf{q}_{0t}^{\Sigma,T}$ follows by replacing σ_t with ς_t. Finally, the capital law of motion is given by (C59) using the island drifts derived above. This completes the proof of Proposition 5.

C.5 Prices and Investment

The estimation in Section 4.4 relies on second-order approximations of wages and investment. Both wages and the rental rate are log-linear in the state. The rental-rate loadings e^R are defined in Appendix C.3, while the wage loadings are $\mathrm{e}^w_{\chi,i} = 1$, $\mathrm{e}^w_{k,i} = \alpha(1-\omega-\varpi)$, and $\mathrm{e}^w_{n,i} = -\alpha(1-\varpi)/N_i^{ss}$. Applying Lemma 4, we obtain:

$$\widehat{w}_t = \sum_x \mathrm{D}_{wx} x + \frac{\epsilon}{2}\sum_{x,y} \mathbb{D}_{wxy} \times_3 y \times_2 x, \qquad \widehat{R}_t = \sum_x \mathrm{D}_{Rx} x + \frac{\epsilon}{2}\sum_{x,y} \mathbb{D}_{Rxy} \times_3 y \times_2 x,$$

where $\widehat{w}_t \equiv (w_t - w^{ss})/\epsilon$ and $\widehat{R}_t \equiv (R_t - R^{ss})/\epsilon$ are the scaled deviations, and:

$$\begin{aligned}\mathrm{D}_{wx} &\equiv \mathbf{diag}\big(w_i^{ss}\, \mathrm{e}^w_{x,i}\big), \\ \mathbb{D}_{wxy} &\equiv \mathbf{diag}_3\big(w_i^{ss} \mathrm{e}^w_{x,i} \mathrm{e}^w_{y,i}\big) + \mathbb{1}_{x=y=n}\mathbf{diag}_3\Big(\frac{w_i^{ss}\alpha(1-\varpi)}{(N_i^{ss})^2}\Big).\end{aligned}$$

The coefficients D_{Rx} and $\mathbb{D}_{Rxy}$ are defined in (C15) and Appendix C.4.

Furthermore, the scaled deviation of investment, $\widehat{I}_t \equiv (I_t - I^{ss})/\epsilon$ with $I_{it} = c_{i0} Q_{it}^{\zeta} K_{it}$, has the second-order expansion:

$$\begin{aligned}\widehat{I}_t &= \mathrm{D}_{I\mathcal{K}} k + \mathrm{D}_{IQ} q_{1t} \\ &\quad + \frac{\epsilon}{2}\Big[\mathbb{D}_{I\mathcal{K}\mathcal{K}} \times_3 k \times_2 k + 2\,\mathbb{D}_{I\mathcal{K}Q} \times_3 q_{1t} \times_2 k + \mathbb{D}_{IQQ} \times_3 q_{1t} \times_2 q_{1t} + \mathrm{D}_{IQ} q_{2t}\Big],\end{aligned}$$

where:

$$\mathbb{D}_{I\mathcal{K}\mathcal{K}} \equiv \mathbf{diag}_3\big(I_i^{ss}\big), \qquad \mathbb{D}_{I\mathcal{K}Q} \equiv \zeta\, \mathbf{diag}_3\Big(\frac{I_i^{ss}}{Q_i^{ss}}\Big), \qquad \mathbb{D}_{IQQ} \equiv \zeta(\zeta-1)\, \mathbf{diag}_3\Big(\frac{I_i^{ss}}{(Q_i^{ss})^2}\Big),$$

with $\mathrm{D}_{I\mathcal{K}} \equiv \mathbf{diag}(I_i^{ss})$, $\mathrm{D}_{IQ} \equiv \mathbf{diag}(K_i^{ss})\,\mathrm{D}_{\mathcal{I}Q}$, and $I_i^{ss} = \Delta_i K_i^{ss}$. Since our estimation uses the diversified economy, we omit the island counterparts. These have the same structure, with the investment rule $g_i(Q_{it}, R_{it})$ replacing $c_{i0}Q_{it}^{\zeta}$.

D Risk Premia and Welfare

This appendix derives the risk premia and welfare measures used in Section 5. We also characterize the no-covariance valuation of local risk.

D.1 Stochastic Discount Factors and Risk Premia

We derive the stochastic discount factors and capital risk premia in both models, allowing for correlated shocks. These covariances also enter the Master Equation expansions of Appendix C.2.

The diversified capitalist. In the diversified economy, capital is priced using the discounted marginal utility of aggregate capitalist consumption, as in Proposition 1:

$$\Lambda_t = e^{-\rho t}\overline{u}'(C_t)/\overline{u}'(C_0) = e^{-\rho t}(C_t/C_0)^{-\overline{\gamma}},$$

with dynamics given by (B21):

$$\frac{d\Lambda_t}{\Lambda_t} = -\overline{R}_t\,dt - \overline{\gamma}\left(dC_t/C_t - \mathbb{E}_t[dC_t/C_t]\right).$$

The bond Euler equation determines the drift $-\overline{R}_t$, while the kernel loadings $-\overline{\gamma}\sigma^C_{z,t}$, $-\overline{\gamma}\sigma^C_{\chi,\ell,t}$, and $-\overline{\gamma}\sigma^C_{\Delta,\ell,t}$ give the market prices of risk for the three families of shocks. Here, σ^C denotes the consumption-growth loadings in (C4).

Let dR^K_{it} denote the return on one unit of installed capital in location i, including the rental flow, investment surplus, and capital gains, net of depreciation. Its expected value is:

$$\frac{1}{dt}\mathbb{E}_t\left[dR^K_{it}\right] = \frac{1}{Q_{it}}\left[R_{it} + \frac{c_{i0}}{1+\zeta}Q_{it}^{1+\zeta} + \mathbb{E}_t\left[\frac{dQ_{it}}{dt}\right] + \frac{1}{dt}\mathrm{Cov}_t\Big(dQ_{it}, \frac{dK_{it}}{K_{it}}\Big)\right] - \Delta_{it}.$$

The return inherits the diffusion loadings of dQ_{it}/Q_{it}, namely $\sigma^Q_{z,i,t}/Q_{it}$, $\sigma^Q_{\chi,i,\ell,t}/Q_{it}$, and $\sigma^Q_{\Delta,i,\ell,t}/Q_{it}$, together with the direct depreciation loading $-\Sigma_{\Delta,it}$ on $dW_{\Delta,it}$. The capital-price loadings σ^Q are defined in Appendix C.2.

Rearranging the pricing equation in Proposition 1, the capital risk premium is the expected

return in excess of the risk-free rate, or equivalently, minus the covariance of the return with the stochastic discount factor:

$$\begin{aligned}\pi_{it} &\equiv \frac{1}{dt}\mathbb{E}_t\big[dR^K_{it}\big] - \overline{R}_t = -\frac{1}{dt}\mathbb{C}\mathrm{ov}_t\Big(\frac{d\Lambda_t}{\Lambda_t}, dR^K_{it}\Big) \\ &= \frac{\overline{\gamma}}{dt}\left[\frac{1}{Q_{it}}\mathbb{C}\mathrm{ov}_t\Big(dQ_{it}, \frac{dC_t}{C_t}\Big) + \mathbb{C}\mathrm{ov}_t\Big(\frac{dK_{it}}{K_{it}}, \frac{dC_t}{C_t}\Big)\right] \\ &= \frac{\overline{\gamma}}{Q_{it}}\Big(\sigma^C_{z,t}\sigma^Q_{z,i,t} + \sum_{\ell,m}\mathfrak{c}_{\chi,\ell m}\,\sigma^C_{\chi,\ell,t}\sigma^Q_{\chi,i,m,t} + \sum_{\ell,m}\mathfrak{c}_{\Delta,\ell m}\,\sigma^C_{\Delta,\ell,t}\sigma^Q_{\Delta,i,m,t}\Big) \\ &\quad - \overline{\gamma}\,\Sigma_{\Delta,it}\sum_{\ell}\mathfrak{c}_{\Delta,i\ell}\,\sigma^C_{\Delta,\ell,t}. \end{aligned} \tag{D1}$$

The first covariance captures the price channel, while the second captures the quantity channel, reflecting the direct destruction of the owner's capital. In terms of diffusion loadings, each component of the risk premium combines the exposure of capital returns to a shock with its corresponding market price of risk. We call the quantity channel the partial equilibrium premium and the remaining terms the general equilibrium component. Section 5.3 reports the no-covariance premium, which removes the covariances between the price of capital and local shocks, as the no-covariance valuation of Appendix D.3 does. In the diversified model, it drops the price-channel terms of the productivity and depreciation shocks in (D1). In the island model, it drops the terms of (D4) that pair the direct loss of the owner's capital in one loading with the response of Q_{it} in the other, through $\bar{c}_{it}$ or q_{it}.

Island capitalists. In the island economy, capitalists hold capital in a single location and cannot trade bonds. Hence, there is no market risk-free rate or market risk premium. Instead, we define their shadow counterparts using each capitalist's stochastic discount factor, given by the discounted marginal utility of own consumption $C^{cap}_{it} = [R_{it} - c_i(g_{it})]K_{it}$:

$$\begin{aligned}\Lambda_{it} &= e^{-\rho t}\frac{\overline{u}'(C^{cap}_{it})}{\overline{u}'(C^{cap}_{i0})} = e^{-\rho t}\left(\frac{C^{cap}_{it}}{C^{cap}_{i0}}\right)^{-\overline{\gamma}}, \\ \frac{d\Lambda_{it}}{\Lambda_{it}} &= -R^f_{it}\,dt - \overline{\gamma}\Big(\sigma^C_{z,i,t}\,dW_t + \sum_{\ell}\sigma^C_{\chi,i,\ell,t}\,dW_{\chi,\ell t} + \sum_{\ell}\sigma^C_{\Delta,i,\ell,t}\,dW_{\Delta,\ell t}\Big), \end{aligned} \tag{D2}$$

where σ^C now denotes the diffusion loadings of $dC^{cap}_{it}/C^{cap}_{it}$.

We write consumption as $C^{cap}_{it} = \bar{c}_{it}K_{it}$, where $\bar{c}_{it} \equiv R_{it} - c_i(g_{it})$ denotes consumption per unit of capital. The notation distinguishes it from workers' consumption c_{it} in Appendix B.1. Consumption responds to depreciation shocks both directly through the capital stock and indirectly

through $\bar{c}_{it}$. Productivity and aggregate shocks, in contrast, affect consumption only through $\bar{c}_{it}$. The corresponding loadings are:

$$\sigma^C_{\Delta,i,\ell,t} = -\Sigma_{\Delta,\ell t}\left(\mathbb{1}_{\{\ell=i\}} + \frac{\partial \log \bar{c}_{it}}{\partial \mathcal{K}_\ell}\right) \quad , \quad \sigma^C_{\chi,i,\ell,t} = \Sigma_{\chi,\ell t}\,\frac{\partial \log \bar{c}_{it}}{\partial \chi_\ell} \quad , \quad \sigma^C_{z,i,t} = \frac{\partial \log \bar{c}_{it}}{\partial z},$$

up to first order. We refer to $\partial \log \bar{c}_{it}/\partial \mathcal{K}_\ell$ as the consumption hedge, the counterpart of the price hedge in Section 2.6.

The shadow risk-free rate is the rate that makes the capitalist indifferent to a marginal position in a riskless asset in zero net supply. From the drift of (D2), we obtain:

$$R^f_{it} = \rho + \overline{\gamma}\,\mu^C_{it} - \frac{\overline{\gamma}(\overline{\gamma}+1)}{2}\,\|\sigma^C_{it}\|^2 \quad , \qquad \mu^C_{it} \equiv \frac{1}{dt}\mathbb{E}_t\left[\frac{dC^{cap}_{it}}{C^{cap}_{it}}\right], \tag{D3}$$

up to second order. The instantaneous variance of consumption growth includes the three families of shocks and their correlations:

$$\begin{aligned}\|\sigma^C_{it}\|^2 = &\sum_{\ell,m}\Sigma_{\Delta,\ell m t}\left(\mathbb{1}_{\{\ell=i\}} + \frac{\partial \log \bar{c}_{it}}{\partial \mathcal{K}_\ell}\right)\left(\mathbb{1}_{\{m=i\}} + \frac{\partial \log \bar{c}_{it}}{\partial \mathcal{K}_m}\right)\\ &+ \sum_{\ell,m}\Sigma_{\chi,\ell m t}\,\frac{\partial \log \bar{c}_{it}}{\partial \chi_\ell}\,\frac{\partial \log \bar{c}_{it}}{\partial \chi_m} + \left(\sigma^C_{z,i,t}\right)^2.\end{aligned}$$

This is the local counterpart of the risk-free rate in Proposition 1.

We next derive the shadow risk premium on a marginal unit of installed capital. As in the diversified economy, let dR^K_{it} denote its return, now using q_{it} as the price of installed capital. The return has direct exposure to capital destruction, $-\Sigma_{\Delta,it}$ on $dW_{\Delta,it}$, as well as the following price loadings: $\sigma^q_{\Delta,i,\ell,t} = -\Sigma_{\Delta,\ell t}\,\partial \log q_{it}/\partial \mathcal{K}_\ell$, $\sigma^q_{\chi,i,\ell,t} = \Sigma_{\chi,\ell t}\,\partial \log q_{it}/\partial \chi_\ell$, and $\sigma^q_{z,i,t} = \partial \log q_{it}/\partial z$. Pricing this return with (D2), the expected excess return over the shadow risk-free rate is:

$$\begin{aligned}\pi_{it} \equiv \frac{1}{dt}\mathbb{E}_t\left[dR^K_{it}\right] - R^f_{it} &= -\frac{1}{dt}\mathbb{C}\mathrm{ov}_t\left(\frac{d\Lambda_{it}}{\Lambda_{it}}, dR^K_{it}\right)\\ &= \overline{\gamma}\sum_{\ell,m}\Sigma_{\Delta,\ell m t}\left(\mathbb{1}_{\{\ell=i\}} + \frac{\partial \log \bar{c}_{it}}{\partial \mathcal{K}_\ell}\right)\left(\mathbb{1}_{\{m=i\}} + \frac{\partial \log q_{it}}{\partial \mathcal{K}_m}\right)\\ &\quad + \overline{\gamma}\sum_{\ell,m}\Sigma_{\chi,\ell m t}\,\frac{\partial \log \bar{c}_{it}}{\partial \chi_\ell}\,\frac{\partial \log q_{it}}{\partial \chi_m} + \overline{\gamma}\,\sigma^C_{z,i,t}\,\sigma^q_{z,i,t},\end{aligned} \tag{D4}$$

up to second order. The three components capture depreciation, productivity, and aggregate-temperature risk. They correspond to the channels in (D1), but use the local capitalist's price of risk rather than the national one. The quantity channel is now $\overline{\gamma}\,\Sigma_{\Delta,iit}$, with $\Sigma_{\Delta,iit} \equiv \Sigma^2_{\Delta,it}$ the own

variance of the destruction shock, which the local owner bears in full.

Finally, we derive the consumption and price hedges from the investment rule $g_{it} = g_i(Q_{it}, R_{it})$. For $x \in \{\mathcal{K}_\ell, \chi_\ell, z\}$, applying the chain rule to $\bar{c}_{it} = R_{it} - c_i(g_{it})$ gives:

$$\frac{\partial \log \bar{c}_{it}}{\partial x} = \frac{1}{\bar{c}_{it}} \Big\{ \big[1 - q_{it}\, g_{i,R}\big] \frac{\partial R_{it}}{\partial x} - q_{it}\, g_{i,Q} \frac{\partial Q_{it}}{\partial x} \Big\},$$

while $q_{it} = c_i'(g_{it})$ and $c_i''(g)/c_i'(g) = 1/(\zeta g)$ imply:

$$\frac{\partial \log q_{it}}{\partial x} = \frac{1}{\zeta\, g_{it}} \left(g_{i,Q} \frac{\partial Q_{it}}{\partial x} + g_{i,R} \frac{\partial R_{it}}{\partial x} \right).$$

At steady state, $g_{it} = \Delta_i$ and $\partial R_{it}/\partial z = 0$. The remaining rental-rate derivatives are:

$$\frac{\partial R_{it}}{\partial \mathcal{K}_\ell} = -\phi R_i^{ss} \mathbb{1}_{\{\ell = i\}} \ , \qquad \frac{\partial R_{it}}{\partial \chi_\ell} = R_i^{ss} \mathbb{1}_{\{\ell = i\}}.$$

The derivatives of Q are given by the first-order loadings $Q^k_{i\ell}$, $Q^\chi_{i\ell}$, and q^z_{it} from Appendix C.3, while the policy derivatives $g_{i,Q}$ and $g_{i,R}$ are given in (C32). We evaluate the risk premia and risk-free rates in both models along the second-order solution path.

D.2 Consumption-Equivalent Welfare

Throughout, dV_{is} denotes the deviation of a worker's value at date s from its reference steady-state value. This reference coincides with the expansion point in the geography exercises but is the initial steady state in the transition exercises, where we expand around several steady states. The same convention applies to capitalists' values.

Workers. We define the consumption equivalent ce_{is} in location i at date s as:

$$\mathbb{E}_s \int_s^\infty e^{-\rho(t-s)} \Big\{ u\big((1 + \text{ce}_{is})\, \widetilde{C}^{ss}_{i(t)}\big) + \epsilon^{ss}_{i(t)t} \Big\} dt = V_i^{ss} + dV_{is},$$

where $\epsilon^{ss}_{i(t)t}$ denotes the preference shock associated with the current location. Let $V_i^{C,ss}$ denote the consumption component of the steady-state value, which satisfies:

$$(\rho + \mu)\, V_i^{C,ss} = u(\widetilde{C}_i^{ss}) + \mu \sum_j m_{ij}^{ss}\, V_j^{C,ss},$$

where m_{ij}^{ss} are the steady-state migration shares. Under CRRA utility with $\gamma \neq 1$, the consumption equivalent is:

$$\text{ce}_{is} = \Big(1 + \frac{dV_{is}}{V_i^{C,ss} + \frac{1}{\rho(1-\gamma)}}\Big)^{\frac{1}{1-\gamma}} - 1. \tag{D5}$$

For log utility, $\text{ce}_{is} = e^{\rho\, dV_{is}} - 1$.

Finally, we aggregate local consumption equivalents using steady-state population and baseline consumption, which equals real income:

$$\overline{\text{ce}}_s \equiv \frac{\sum_i N_i^{ss} \frac{(1+s)\, w_i^{ss}}{(r_i^{ss})^{\beta}} \text{ce}_{is}}{\sum_i N_i^{ss} \frac{(1+s)\, w_i^{ss}}{(r_i^{ss})^{\beta}}}.$$

Diversified capitalist. The diversified capitalist holds capital across all locations, so welfare is measured nationally. The consumption equivalent ce_s^{cap} satisfies:

$$\int_s^{\infty} e^{-\rho(t-s)}\, \overline{u}\big((1 + \text{ce}_s^{cap})\, C^{ss}\big)\, dt = \mathcal{W}_s^{cap},$$

where $\mathcal{W}_s^{cap} \equiv \mathbb{E}_s \int_s^{\infty} e^{-\rho(t-s)}\, \overline{u}(C_t)\, dt$ denotes lifetime utility along the counterfactual path. Solving for ce_s^{cap} gives:

$$\text{ce}_s^{cap} = \left[\frac{\rho(1-\overline{\gamma})\, \mathcal{W}_s^{cap} + 1}{\rho(1-\overline{\gamma})\, \mathcal{W}^{cap,ss} + 1}\right]^{\frac{1}{1-\overline{\gamma}}} - 1.$$

To express this in terms of the model's equilibrium objects, let $\Omega_s \equiv \sum_i Q_{is} K_{is}$ denote the value of the capitalist's portfolio. Using the homogeneity of the value function of degree $1-\overline{\gamma}$ in capital stocks and the costate identity from Appendix B.2, we obtain:

$$\rho(1-\overline{\gamma})\, \mathcal{W}_s^{cap} + 1 = \rho\, C_s^{-\overline{\gamma}}\, \Omega_s, \qquad \rho(1-\overline{\gamma})\, \mathcal{W}^{cap,ss} + 1 = \rho\, (C^{ss})^{-\overline{\gamma}}\, \Omega^{ss}.$$

Substituting these expressions yields:

$$\text{ce}_s^{cap} = \Big(\frac{C_s}{C^{ss}}\Big)^{-\frac{\overline{\gamma}}{1-\overline{\gamma}}} \Big(\frac{\Omega_s}{\Omega^{ss}}\Big)^{\frac{1}{1-\overline{\gamma}}} - 1. \tag{D6}$$

This expression holds for $\overline{\gamma} \neq 1$. In the transition exercises, we instead compute ce_s^{cap} from the certainty-equivalent value $\mathcal{J}$ of Appendix H.1.2, with $\overline{\varphi}^{-1} = \overline{\gamma}$, using (H20). Formula (D6) com-

bines C_s and Ω_s, which are approximated separately. The value $\mathcal{J}$ is approximated directly and is continuous across expansions.

Island capitalists. The island capitalist in location i has the value function $\mathcal{Q}_{is}(K_{is})$ defined in (B25). The consumption equivalent ce_{is}^{cap} satisfies:

$$\int_s^\infty e^{-\rho(t-s)}\, \overline{u}\big((1+\text{ce}_{is}^{cap})\, C_i^{cap,ss}\big)\, dt = \mathcal{Q}_{is}(K_{is}),$$

which gives:

$$\text{ce}_{is}^{cap} = \left[\frac{\rho(1-\overline{\gamma})\, \mathcal{Q}_{is}(K_{is})+1}{\rho(1-\overline{\gamma})\, \mathcal{Q}_i^{ss}(K_i^{ss})+1}\right]^{\frac{1}{1-\overline{\gamma}}} - 1.$$

Using (B25), we have $\rho(1-\overline{\gamma})\, \mathcal{Q}_{is}(K_{is})+1 = \rho\,(Q_{is}K_{is})^{1-\overline{\gamma}}$. Substituting this expression yields:

$$\text{ce}_{is}^{cap} = \frac{Q_{is}K_{is}}{Q_i^{ss}K_i^{ss}} - 1, \tag{D7}$$

which is the proportional change in certainty-equivalent wealth $Q_{is}K_{is}$. To aggregate across locations, we weight each consumption equivalent by steady-state capitalist consumption $C_i^{cap,ss}$, which is proportional to $Q_i^{ss}K_i^{ss}$:

$$\overline{\text{ce}}_s^{cap} \equiv \frac{\sum_i C_i^{cap,ss}\, \text{ce}_{is}^{cap}}{\sum_i C_i^{cap,ss}} = \frac{\sum_i Q_{is}K_{is}}{\sum_i Q_i^{ss}K_i^{ss}} - 1.$$

D.3 General Equilibrium and No-Covariance Valuation of Local Risk

In the no-covariance exercises of Section 5.1, we remove the covariances between capital prices and local shocks from the capitalists' local-risk forcings in the SAME. For the diversified economy, this gives:

$$\mathbf{q}_{0t}^{\Sigma,z}\big|_{\text{no cov.}} \equiv \mathbf{q}_{0t}^{\Sigma,z} - \Big[\mathrm{d}_{Q\Sigma K} + \big(\mathrm{D}_{Q\Sigma\chi} + \mathrm{D}_{Q\Sigma\Delta}\big)\,\mathbb{1}\Big]\sigma_t,$$

while for the island economy:

$$\mathbf{q}_{0t}^{\Sigma,z}\big|_{\text{no cov.}} \equiv \mathbf{q}_{0t}^{\Sigma,z} - \mathrm{d}_{Q\Sigma K}\,\sigma_t.$$

In both cases, $\mathbf{q}_{0t}^{\Sigma,T}$ follows by replacing σ_t with ς_t. All other forcings, including those of the constant block, as well as the worker equations and laws of motion, remain unchanged. Thus, the exercise retains the first-order response of capital prices while removing only their second-order covariances with local shocks. In the island model, the exercise keeps the variance of the value of capital, $\mathrm{d}_{Q\Sigma Q}$: it removes the hedge, not the risk that price fluctuations create. We apply this adjustment either in the stochastic steady state or along the transition, depending on the exercise.

Since the Σ block is linear in its forcings, the general equilibrium valuation can be decomposed into the no-covariance valuation and the contribution of the excluded covariances.

E Data and Measurement

E.1 Economic Data

Our economic dataset combines county-year information from several sources. Population data come from the BEA, which constructs annual county population estimates by interpolating between decennial Census counts and later replaces them with official intercensal figures as those become available. For income, we use BEA personal income per capita, a broad measure that includes labor income, transfers, employer contributions, interest income, and various imputed components. To capture labor earnings more directly, we also use BEA salary income. Employment is likewise taken from the BEA and reflects both full-time and part-time jobs.

To measure investment, we use county-level manufacturing capital expenditures from the Census of Manufactures for 1958 to 1992 and from the Economic Censuses for 1997 to 2012. The earlier Census of Manufactures reports are available through the ICPSR 2896 digitization project, while the later Economic Census data are released at five-year intervals. Because these investment data are not observed annually, we linearly interpolate values between Census years before estimating the event studies. Finally, we collect data on local public spending from the Government Finance Database, which harmonizes information from the Census of Governments. We use total local government expenditures as a control variable and interpolate these data between Census years as well.

E.2 ISIMIP3a Data

Our climate data come from the Inter-Sectoral Impact Model Intercomparison Project (ISIMIP), which develops datasets designed to study the effects of climate change on both natural and socioeconomic systems. We use the high-frequency observed climate dataset from ISIMIP simulation

round 3a, which covers 1901 to 2019 on a $0.5^\circ \times 0.5^\circ$ grid. This product combines two underlying datasets: the Global Soil Wetness Project Phase 3 (GSWP3) for 1901 to 1978 (Kim, 2017), and the WATCH Forcing Data methodology applied to the ERA5 reanalysis dataset (W5E5) for 1979 to 2019 (Lange et al., 2021). Both datasets are constructed by blending numerical weather prediction models with station-based atmospheric observations (Mengel et al., 2021).

The historical GSWP3 component ultimately builds on the Twentieth Century Reanalysis Project (20CR), which spans 1871 to 2008 at a 2° spatial resolution and 6-hour frequency. In 20CR, atmospheric conditions are first generated using NOAA's NCEP Global Forecast System and then updated using observed and derived surface and sea-level pressure data through an Ensemble Kalman Filter assimilation procedure (Compo et al., 2011; Cucchi et al., 2020). The lower-boundary inputs for these simulations are based on sea surface temperature and sea-ice concentration from the UK Met Office HadISST1.1 dataset (Rayner et al., 2003), interpolated from monthly to daily frequency. The pressure observations used in the assimilation stage come from version 2 of the International Surface Pressure Databank (Cram et al., 2015), which compiles records from a wide range of meteorological agencies.

GSWP3 is obtained from 20CR through dynamic downscaling and bias correction of selected variables. The original 2° fields are first downscaled to roughly 0.5° resolution for 1901 to 2010 at a 3-hour frequency using spectral nudging within a Global Spectral Model (Yoshimura and Kanamitsu, 2008). The resulting series are then bilinearly interpolated to a regular 0.5° grid and further bias-adjusted using observational reference datasets. In particular, precipitation is corrected using the Global Precipitation Climatology Centre Full Data Monthly Product Version 7 (Schneider et al., 2014), wind speed using the Climatic Research Unit CL2.0 dataset (New et al., 2002), and temperature, pressure, and humidity using the Climatic Research Unit TS3.23 dataset (Harris et al., 2014).

The more recent W5E5 component is based on the European Centre for Medium-Range Weather Forecasts (ECMWF) reanalysis dataset ERA5 (Hersbach et al., 2020). ERA5 begins with numerical weather predictions generated by the ECMWF Integrated Forecasting System Cycle 41r2 and then assimilates observational data through a two-part procedure: an incremental 4D-Var method for atmospheric variables and ozone (Courtier et al., 1994), and a land data assimilation system that combines optimal interpolation methods for temperature, relative humidity, and snow with a Simplified Extended Kalman Filter for soil moisture. Across 1979 to 2019, this assimilation system incorporates roughly 96 billion observations from a wide range of conventional and satellite sources worldwide. The resulting ERA5 product has hourly frequency and a spatial resolution of

about 31 kilometers.

To construct the forcing dataset used in ISIMIP, ERA5 is then bias-corrected and aggregated to a 0.5° grid following the WATCH framework for land-surface and global hydrological modeling. This produces the WFDE5 dataset (Cucchi et al., 2020). The bias adjustments are designed to align monthly moments with observed data, using the Climatic Research Unit TS4.03 dataset (Harris et al., 2020) for surface air temperature, shortwave radiation, rainfall, and snowfall, and the Global Precipitation Climatology Centre Full Data Monthly Product Version 2018 (Schneider et al., 2018) for rainfall and snowfall. W5E5, the ISIMIP input dataset, is then formed by combining WFDE5 over land with ERA5 over the ocean (Lange et al., 2021).

Because the older 20CR-based data and the bias-correction procedures underlying GSWP3 are known to contain artifacts and spurious trends, and because ERA5 relies on a much larger observational base, the ISIMIP3a authors view W5E5 as the more realistic dataset. To extend W5E5 back to 1901, the earlier GSWP3 series are homogenized to ensure a smooth transition at the 1978–1979 break. Specifically, over the common 1979 to 2004 reference period, GSWP3 is quantile-mapped to the corresponding W5E5 distributions (Lange, 2019; Lange, 2021). The final GSWP3-W5E5 dataset therefore preserves the long-run trends present in GSWP3, coincides exactly with W5E5 from 1979 onward, and differs from the original GSWP3 series for 1901 to 1978.

E.3 Event Studies and Event Frequencies

Figure E1 reproduces the event studies of Bilal and Rossi-Hansberg (2025) whose ten-year cumulative responses we target in Sections 4.4 and 4.5. A 1-in-50-year storm in a coastal county lowers income per capita by about 2.5% on average over the following decade, reaching about 3% at its end. Employment falls by roughly 5% after ten years. Investment rises sharply and peaks at around 20% two to three years after the storm. This pattern is the response to a capital depreciation shock. A 1-in-20-year heat wave in a warm county lowers income per capita by about 1.5% and leaves it persistently depressed. Employment falls by up to 1.5% in the years after the event. Investment falls moderately. This pattern is the response to a productivity loss.

Figure E2 shows how the frequency of the two events has moved with global mean temperature, the relationship that Section 4.5 estimates location by location. The annual probability of a 1-in-50-year storm rises by 0.069 per degree of warming in coastal counties and by only 0.012 per degree inland. The annual probability of a 1-in-20-year heat wave rises by 0.052 per degree in warm counties and falls by 0.033 per degree in cold counties.

E.4 Spatial Correlations

Section 4.5 estimates the correlations $\mathfrak{c}_{ij}$ of (A2) and (A4) using yearly event indicators for 100 clusters of counties. We correlate the event indicators after removing their temperature trends. Let e_{iy} be the average residual across the counties of cluster i in year y from regression (18). The cluster-level correlation is:

$$\mathfrak{C}_{ij} = \frac{\sum_y e_{iy} e_{jy}}{\sqrt{\sum_y e_{iy}^2 \sum_y e_{jy}^2}}.$$

We compute these correlations over 1920 to 2019. The resulting matrix is symmetric and positive semi-definite, with unit diagonal. Under the Poisson model with yearly counts, $\mathbb{C}\text{ov}(n_{iy}, n_{jy} \mid T) = \mathfrak{c}_{ij}\sqrt{p_{iy}p_{jy}}$ and $\mathbb{V}\text{ar}(n_{iy} \mid T) = p_{iy}$. Thus, $\mathfrak{C}_{ij}$ estimates $\mathfrak{c}_{ij}$ up to a factor that equals one when the arrival rates have proportional trends.

The model is solved at the county level. Counties in the same cluster are assigned correlation $\mathfrak{c}_w$. We estimate it as the correlation of the residuals of regression (18), pooled over all pairs of counties in the same cluster. The estimates are 0.27 for storms and 0.55 for heat waves. Counties in different clusters i and j are assigned correlation $\mathfrak{c}_{ij} = \mathfrak{C}_{ij}\sqrt{\widetilde{v}_i \widetilde{v}_j}$. Here, $\widetilde{v}_i = [1 + (\mathcal{I}_i - 1)\mathfrak{c}_w]/\mathcal{I}_i \leq 1$ is the variance of the fraction of counties hit in cluster i, relative to that of a single county, and $\mathcal{I}_i$ is the number of counties in the cluster. The adjustment accounts for averaging within clusters and the resulting matrix is positive semi-definite whenever $\mathfrak{C}$ is.

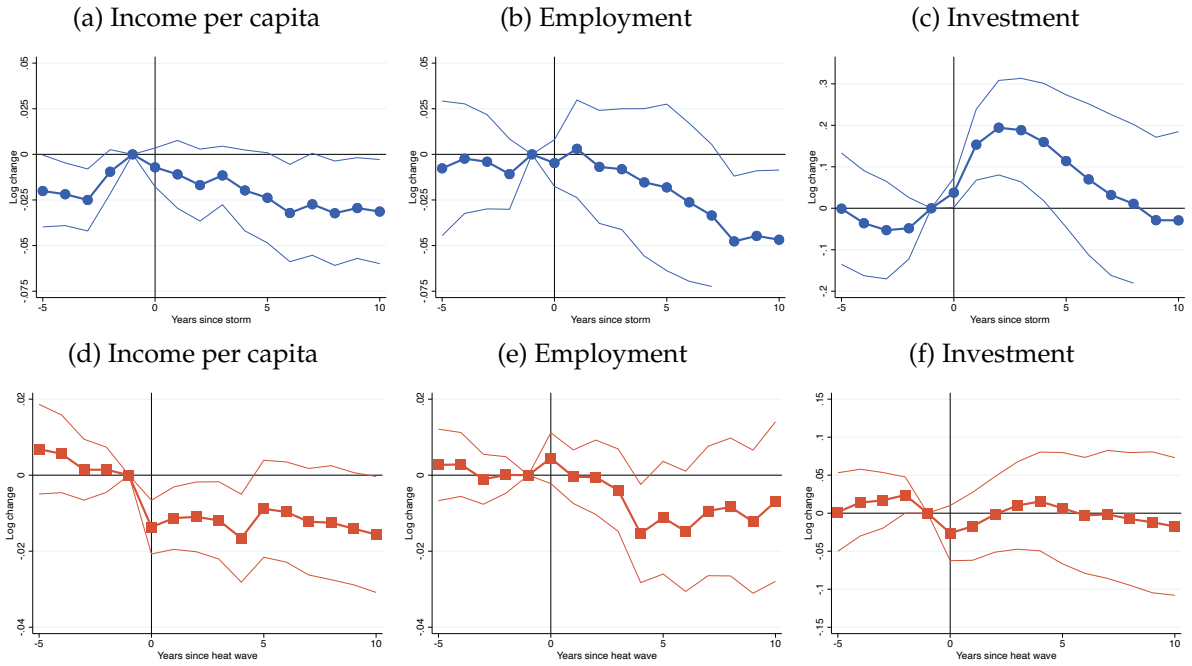
F Steady-State Inversion

We prove Proposition 6 by construction, following the same inversion steps in both models. Productivity is at its long-run value, so its drift is zero. We fix baseline depreciation rates Δ_i and use data on wages w_i, population N_i, investment I_i, and migration flows $\mathbb{m}_{ij}$.

Steady-state capital accumulation, $g_i^{ss} = \Delta_i$ in (14), gives the capital stock $K_i = I_i/\Delta_i$. We recover the stocks of buildings B_i and housing H_i using the constant shares (B1)-(B2) and the buildings production function:

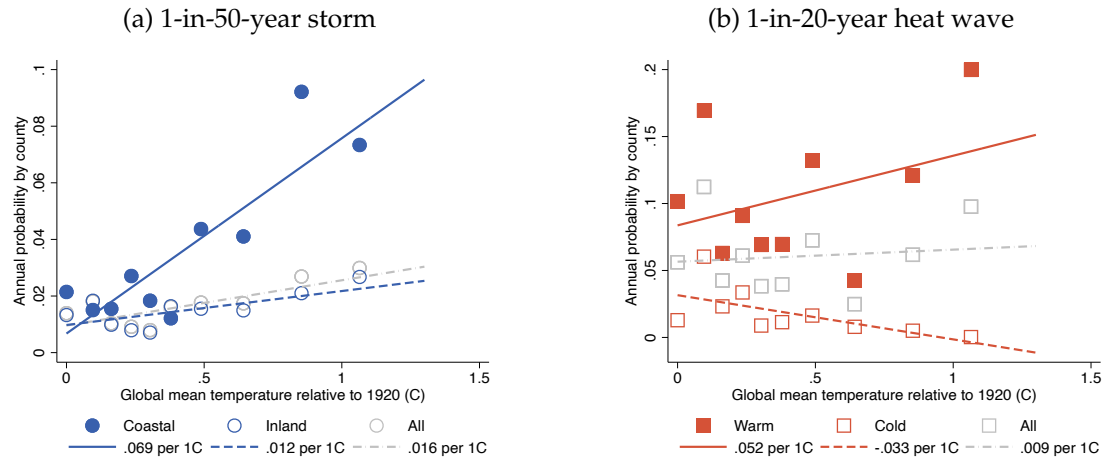
$$B_i = L_i^{\omega} \Big((1 - \varkappa^N) N_i \Big)^{\varpi} K_i^{1-\omega-\varpi} \quad , \quad H_i = \varkappa^H B_i.$$

Figure E1: Event studies of storms and heat waves.



Note: Reproduced from Bilal and Rossi-Hansberg (2025). Top row, panels (a) to (c): effect of a 1-in-50-year storm in coastal counties. Bottom row, panels (d) to (f): effect of a 1-in-20-year heat wave in warm counties. Log change in income per capita, employment and investment by year relative to the event, normalized to zero the year before the event. Thin lines: 95% confidence bands.

Figure E2: Frequency of extreme events and global temperature.



Note: Reproduced from Bilal and Rossi-Hansberg (2025). Annual probability of the event by county, for each group of counties, against global mean temperature relative to 1920, with a fitted line by group. Panel (a): coastal, inland and all counties. Panel (b): warm, cold and all counties. The legends report the slope of each line per 1°C.

The wage equation (B3) then delivers fundamental productivity:

$$Z_i = \frac{w_i}{(1-\alpha)\alpha^{\alpha}\Xi^{-\alpha}\left(B_i/N_i\right)^{\alpha}},$$

from which we construct $c_{0i}^{w} = (1+s)(1-\alpha)\alpha^{\alpha(1-\beta)}\Xi^{-\xi}(1-\varkappa^{N})^{\omega\xi}Z_i^{1-\beta}L_i^{\omega\xi}$, $R_{0i} = (1-\omega-\varpi)\alpha^{\alpha}\Xi^{1-\alpha}(1-\varkappa^{N})^{\alpha\varpi}Z_iL_i^{\omega\alpha}$, and the steady-state rental rate $R_i = R_{0i}K_i^{-\phi}N_i^{\psi}$.

To recover the capital adjustment cost shifters, we use (14):

$$c_i'(\Delta_i) = \frac{R_i}{\rho + \frac{\zeta}{1+\zeta}\Delta_i} \;\;,\;\; c_{i0} = \frac{\Delta_i}{c_i'(\Delta_i)^{\zeta}} = \Delta_i\left[\frac{\rho + \frac{\zeta}{1+\zeta}\Delta_i}{R_i}\right]^{\zeta}, \tag{F1}$$

where we use $c_i(\Delta_i) = \frac{\zeta}{1+\zeta}\Delta_i c_i'(\Delta_i)$ from (3).

To recover migration costs, we construct migration shares $m_{ij} = \mathtt{m}_{ij}/(\mu\,\mathtt{n}_i)$ for $j \neq i$, where $\mathtt{n}_i$ is the population of i, and $m_{ii} = 1 - \sum_{j\neq i} m_{ij}$, since by (8) the annual flow from i to j is $\mu\, m_{ij}N_i$. Under the normalization $\tau_{ii} = 0$, the migration shares (7) imply $\widetilde{m}_{ij} \equiv \log m_{ij} - \log m_{ii} = \nu(V_j - V_i) - \nu\tau_{ij}$. Combining bilateral migration shares eliminates the values, giving $\widetilde{m}_{ij} + \widetilde{m}_{ji} = -\nu(\tau_{ij} + \tau_{ji})$. Assuming symmetric migration costs, $\tau_{ij} = \tau_{ji}$, we recover:

$$\tau_{ij} = -\frac{\widetilde{m}_{ij} + \widetilde{m}_{ji}}{2\nu}.$$

Finally, we use the steady-state population equation to recover local amenities. Defining $\widetilde{m}_i = e^{\nu V_i}$, the population equation (13), together with a normalization of values, implies:

$$\widetilde{m}_i = \frac{N_i}{\sum_k \frac{N_k\, e^{-\nu\tau_{ki}}}{\sum_j e^{-\nu\tau_{kj}}\widetilde{m}_j}} \;\;,\;\; \sum_i \widetilde{m}_i = 1. \tag{F2}$$

The system (F2) satisfies the gross substitutes property and thus has at most one solution. We solve it iteratively and recover $V_i = \frac{1}{\nu}\log\widetilde{m}_i$. We then invert the steady-state value equation (12) to recover flow utility, $U_i = \rho V_i - \mathcal{M}_i[V]$. The population equation identifies values, and hence flow utilities, only up to an additive constant. We fix it by imposing $\sum_i N_iU_i = 0$, which implies $\sum_i N_i\widetilde{C}_i^{1-\gamma} = 1$ for every γ. Amenities are then recovered as:

$$A_i = \frac{u^{-1}(U_i)}{c_{0i}^{w}K_i^{(1-\omega-\varpi)\xi}N_i^{-(1-\varpi)\xi}}.$$

G Additional Results

This appendix reports additional results for Section 5. The figures correspond to the two exercises of Section 5.1: the geography of risk around the deterministic steady state of a 2°C warmer climate and the transition from 2025 to 2100. All figures use 3,037 counties, except the comparative statics of Figures G9 and G10, which aggregate counties into 1,000 clusters. The figures are collected at the end of this appendix.

Capital and population in the island model. Figure G1 shows how risk affects capital and population in the island model. Local risk reduces capital by up to 17% and population by up to 11% in the most exposed coastal counties. Aggregate risk has the opposite effect, raising coastal capital by up to 5.3% and population by up to 3.2%.

Risk in the diversified model. Diversification greatly reduces capital risk premia. Figure G2 maps these premia across counties under local and aggregate risk. Under local risk, they are about 100 times smaller than in the island model, and they remain positive even inland, where they are essentially zero in the island model.

Despite these lower premia, local risk still drives capital out of the most exposed coastal counties. Figure G3 shows that these counties lose up to 12% of their capital and 7% of their population in the diversified model. Storms destroy exposed capital regardless of who owns it. Under aggregate risk, coastal capital instead rises by up to 2.9% and population by up to 2.4%.

Figure G4 reports worker welfare and changes in the value of local capital under diversification. Worker welfare follows the same coastal gradient as in the island model. Under local risk, local capital value falls by up to 10% along the Gulf and southeast Atlantic coasts. Under aggregate risk, it rises by up to 2.9% on the North Carolina coast.

Capital and population with and without the price covariance. Figure G5 compares changes in capital and population with and without the price covariance. In general equilibrium, all risk reduces coastal capital in 2100 by 1.0% in the island model and 1.3% in the diversified model. Coastal population falls by 0.5% and 0.7%, respectively. Without the price covariance, island capitalists build buffer stocks in their only asset, raising coastal capital by 17% and population by 6%. In the diversified model, removing the covariance changes coastal capital and population by about one percentage point, since diversification removes the precautionary motive.

Dispersion of fundamentals. We next examine how greater dispersion in fundamentals affects the cost of risk. Holding everything else fixed, we scale the deviations of log productivity, amenities, and investment-cost shifters from their means so that each variance doubles. We then introduce all risk at 2°C, as in the geography exercise. Figures G6 and G7 report the results for both models, using capital and consumption weights from each case's 2°C steady state.

Greater dispersion raises capital risk premia. The coastal premium rises from 0.3 to 0.4 p.p. per year in the island model and from 0.004 to 0.01 in the diversified model. The most exposed coastal counties are also among the most productive today. Greater dispersion concentrates more capital in these counties, making alternative locations less attractive for capital and workers.

Workers bear a higher cost of risk. In the island model, coastal welfare losses rise from 0.02 to 0.1%, and inland losses from essentially zero to 0.1%. In the diversified model, worker losses rise by about 60%. These losses reflect reduced insurance provided by migration when alternative locations become less attractive. The effects on capitalists differ across models. Under diversification, the value of coastal capital falls by 2.0% instead of 1.7%, while coastal island capitalists lose slightly less, 3.1% instead of 3.5%.

Specific locations along the transition. Figure G8 illustrates the transition in two counties: Charleston County, South Carolina, which is heavily exposed to climate damages, and Hennepin County, Minnesota, which contains Minneapolis and gains economic activity. The trend dominates both paths. In Charleston, capital falls by about 20% within five years in the diversified model and keeps declining as population leaves. In the island model, it first rises by 5 to 7% by 2030, as local owners save more when their prospects worsen, and then declines. By 2100, Charleston has lost most of its capital and population. Adding aggregate risk, or both local and aggregate risk, shifts the paths of capital and population by at most three percentage points.

In contrast, Minneapolis gains 4 to 9% of capital and 8 to 9% of capitalist wealth. The incidence of losses differs across agents. Capitalists bear the local loss, with wealth in Charleston falling by about 90%. Workers lose less, but the losses spread across counties through migration. By 2100, their lifetime consumption falls by 6% in Charleston and 3% in Minneapolis. The two models imply similar long-run locations of activity but differ in the speed of capital adjustment and the distribution of losses during the transition.

Risk aversion robustness. Finally, we examine how the results change with risk aversion. Figures G9 and G10 vary capitalist and worker risk aversion one at a time, holding the other at its

baseline value. For each value, we re-estimate the elasticities and damages and solve both models using 1,000 county clusters and four expansion points along the transition. The figures report welfare at the initial date of the 2°C transition, with the trend alone and with the trend and all risk. They also report capital in 2100, and risk premia and worker welfare in the geography exercise.

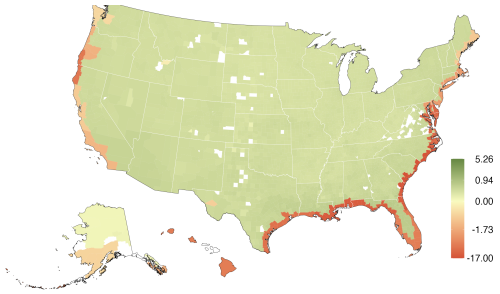
Figure G9 varies capitalist risk aversion from 0.5 to 10. The re-estimated parameters change little, but the cost of risk for island capitalists rises sharply with $\overline{\gamma}$, exceeding the cost of the trend above a risk aversion of about 8 (panel (b)). Thus, our finding that risk is less costly than the trend holds except at very high risk aversion. The diversified capitalist's cost of risk remains negligible throughout. Its risk premium remains about fifty times smaller than the island premium, which rises from 0.03 to 0.1 p.p. per year (panel (d)).

Higher capitalist risk aversion also strengthens precautionary investment. Once $\overline{\gamma}$ exceeds about 4, capital in 2100 with risk exceeds its trend-only level in the island model (panel (c)). Workers in the island model begin to gain from risk above $\overline{\gamma} = 3$, with gains reaching 0.4% at $\overline{\gamma} = 10$. In contrast, workers in the diversified model lose about 0.1% (panel (e)). More risk-averse island capitalists build larger buffer stocks of capital, benefiting workers.

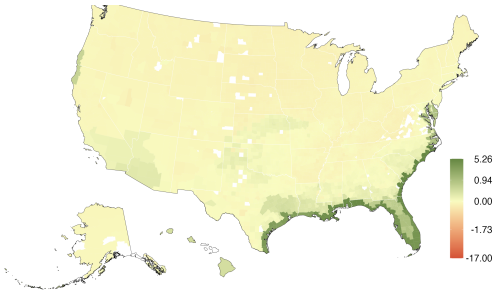
Figure G10 varies worker risk aversion from 1 to 9.5. Unlike for capitalist risk aversion, the re-estimated migration elasticity falls from 0.9 to 0.6 as γ increases. The results therefore reflect both greater risk aversion and lower estimated mobility. Workers' initial welfare losses from the trend barely change: they are 1.6% at $\gamma = 1$ and 1.5% at $\gamma = 9.5$ in the island model, and 1.6% at both values in the diversified model. Risk adds at most 0.1 percentage point to these losses, at $\gamma = 9.5$ (panel (a)). In the geography exercise, workers' cost of risk reaches about 0.2% in both models (panel (e)). Capitalist welfare, capital accumulation, and risk premia change little (panels (b) to (d)). Even at $\gamma = 9.5$, the additional welfare loss of workers from risk is less than one-tenth of their loss from the trend.

Figure G1: Capital and population effects of local and aggregate risk (island model).

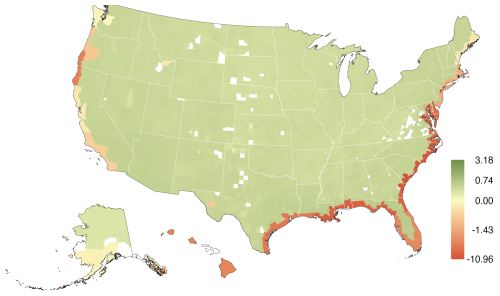
(a) Local risk: capital (%)



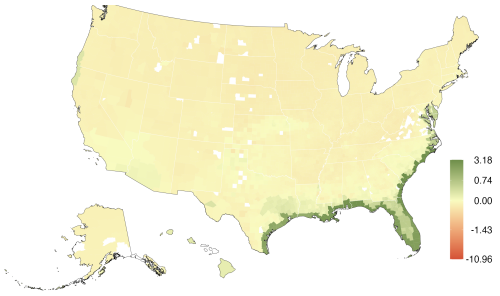
(b) Aggregate risk: capital (%)



(c) Local risk: population (%)



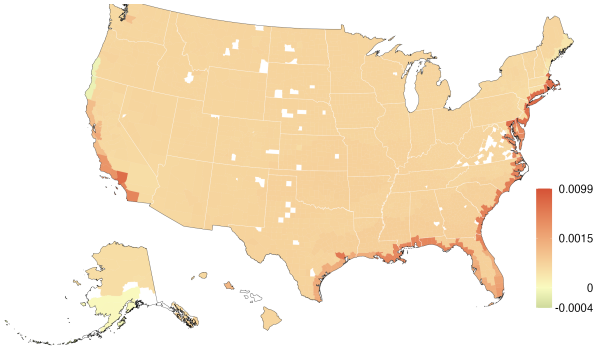
(d) Aggregate risk: population (%)



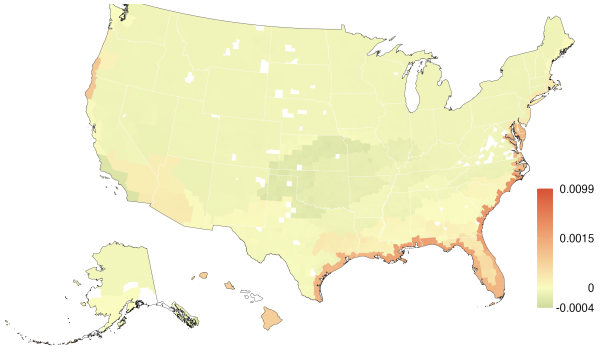
Note: Island model. Change in the capital stock (top row) and in population (bottom row), in percent, in the stochastic steady state with the local risk (left) or the aggregate risk (right) of a 2°C warmer climate, relative to the deterministic steady state of that climate. Each row shares its color scale.

Figure G2: Capital risk premium from local and aggregate risk (diversified model).

(a) Local risk: capital risk premium (% per year)



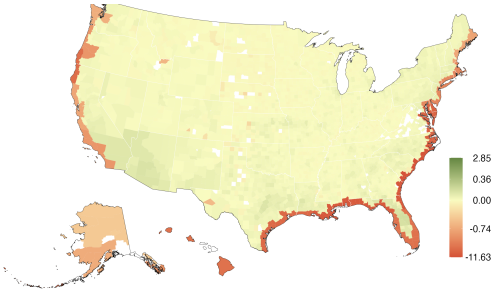
(b) Aggregate risk: capital risk premium (% per year)



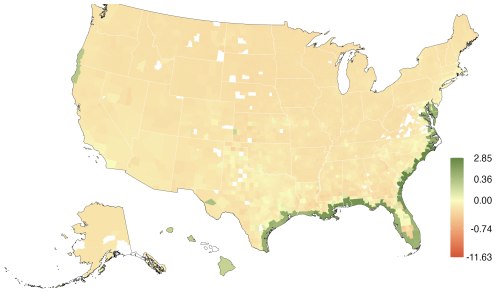
Note: Diversified model. Capital risk premium, in percent per year, in the stochastic steady state with the local risk (a) or the aggregate risk (b) of a 2°C warmer climate, relative to the deterministic steady state of that climate. The two panels share the color scale.

Figure G3: Capital and population effects of local and aggregate risk (diversified model).

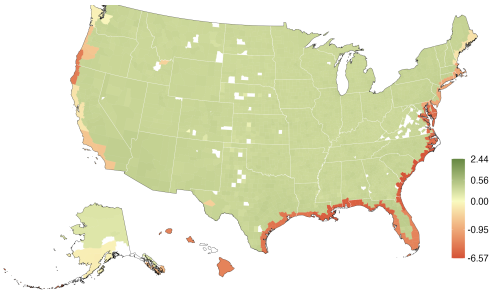
(a) Local risk: capital (%)



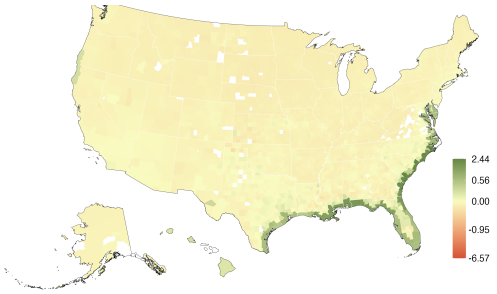
(b) Aggregate risk: capital (%)



(c) Local risk: population (%)



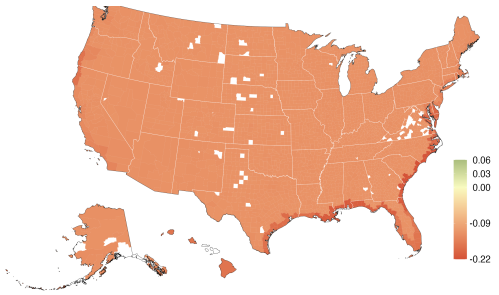
(d) Aggregate risk: population (%)



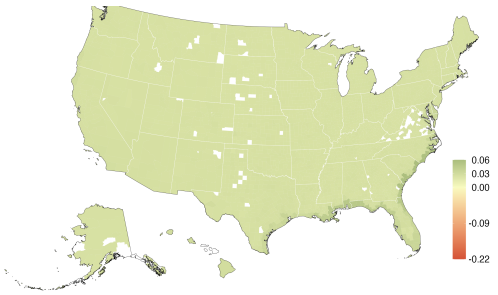
Note: Diversified model. Change in the capital stock (top row) and in population (bottom row), in percent, in the stochastic steady state with the local risk (left) or the aggregate risk (right) of a 2°C warmer climate, relative to the deterministic steady state of that climate. Each row shares its color scale.

Figure G4: Welfare effects of local and aggregate risk (diversified model).

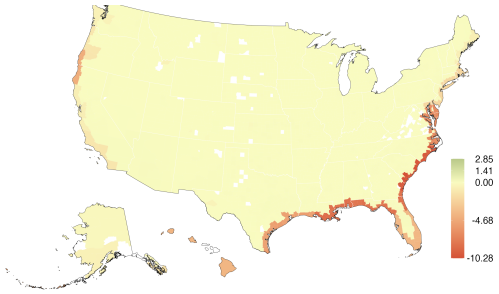
(a) Local risk: worker welfare (% cons.)



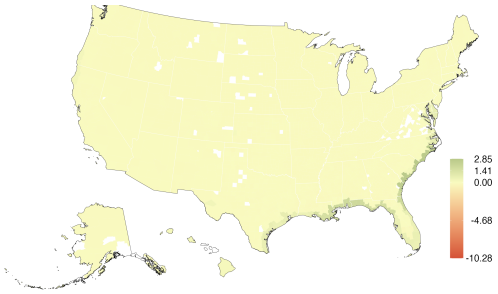
(b) Aggregate risk: worker welfare (% cons.)



(c) Local risk: value of local capital (%)



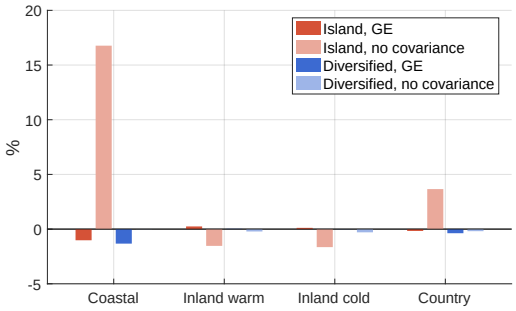
(d) Aggregate risk: value of local capital (%)



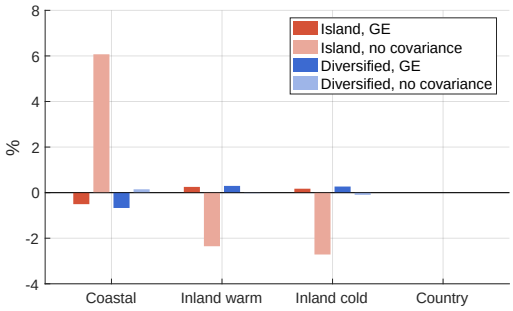
Note: Diversified model. Lifetime consumption-equivalent welfare change of workers (top row) and change in the value of local capital, $Q_{it}K_{it}$ (bottom row), in percent, in the stochastic steady state with the local risk (left) or the aggregate risk (right) of a 2°C warmer climate, relative to the deterministic steady state of that climate. Each row shares its color scale.

Figure G5: Capital and population with and without the price covariance, by location group.

(a) Capital (%)

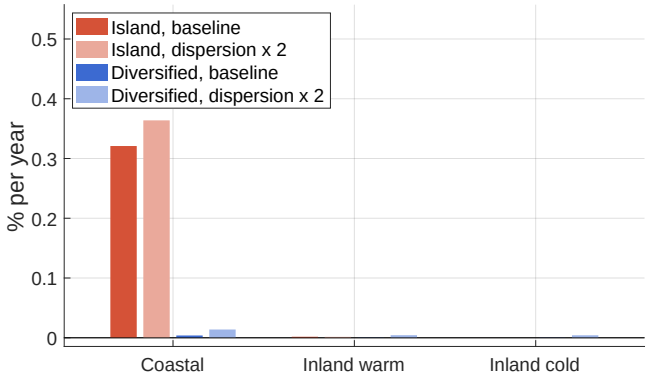


(b) Population (%)



Note: Effect of all risk in 2100 along the 2°C transition: the outcome with the trend and all risk minus the outcome with the trend only. Panel (a): capital stock, in percent of the group's stock. Panel (b): population, in percent of the group's population. Island model in red, diversified model in blue. Dark bars: general equilibrium. Light bars ('no covariance'): the valuation without the covariance between the price of capital and local shocks (Section 5.1). Groups as in Figure 7.

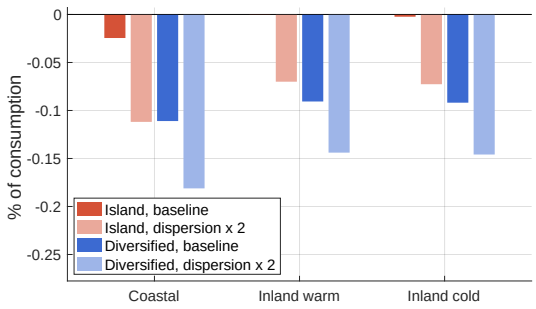
Figure G6: Capital risk premium with more dispersed fundamentals, by location group.



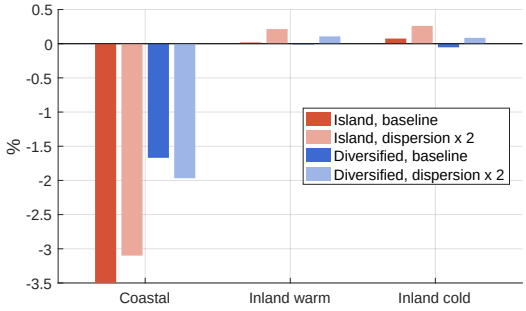
Note: Capital risk premium in the stochastic steady state with the local and aggregate risk of a 2°C warmer climate, in percent per year, capital-weighted within each group. Island model in red, diversified model in blue. Dark bars: baseline fundamentals. Light bars ('dispersion $\times$ 2'): the variance of the log productivity, amenity and investment-cost shifters across counties doubled, everything else unchanged.

Figure G7: Welfare cost of risk with more dispersed fundamentals, by location group.

(a) Worker welfare (% cons.)

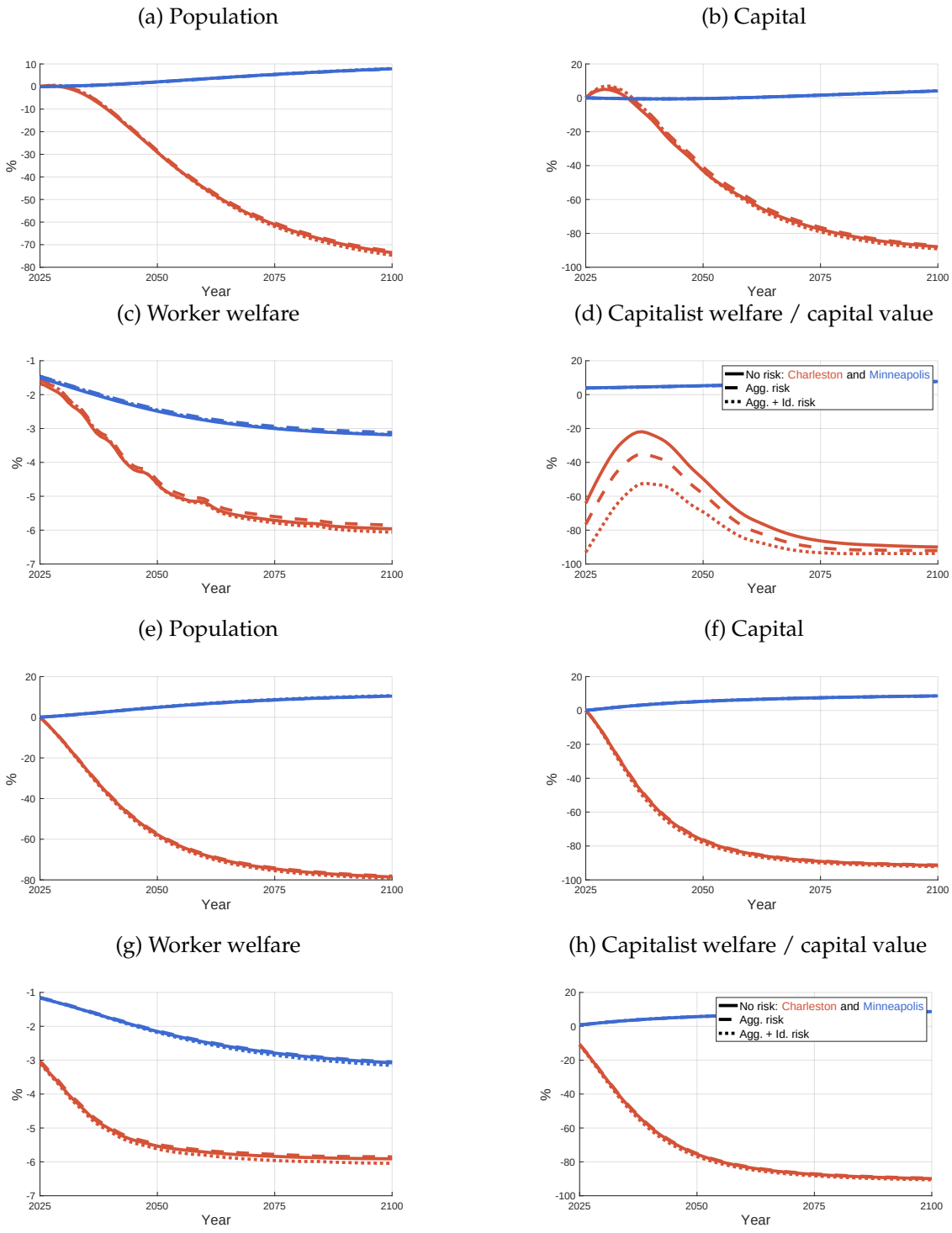


(b) Capitalist welfare / capital value (%)



Note: Welfare effect of the local and aggregate risk of a 2°C warmer climate in the stochastic steady state, in percent. Panel (a): workers, weighted by consumption and population. Panel (b): island capitalists in the island model, weighted by consumption, and the value of local capital in the diversified model, weighted by that value. Island model in red, diversified model in blue. Dark and light bars as in Figure G6.

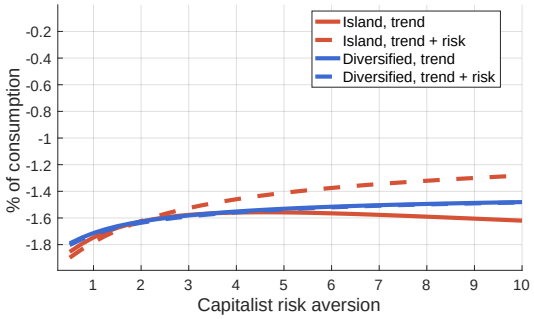
Figure G8: Risk and trend by location: Charleston and Minneapolis.



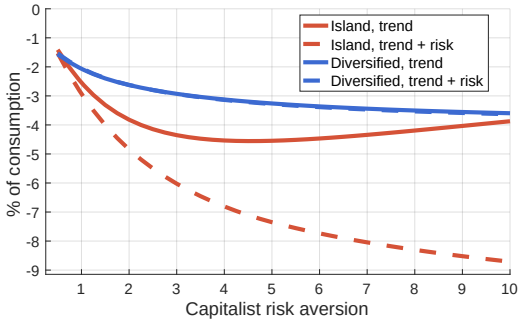
Note: Transition to a 2°C warmer climate by 2100 in Charleston County, South Carolina (red) and Hennepin County, Minnesota (blue). Solid lines: trend only. Dashed lines: trend with aggregate risk. Dotted lines: trend with local and aggregate risk. Panels (a) to (d): island model. Panels (e) to (h): diversified model. Population and capital: change relative to 2025, in percent. Worker welfare: lifetime consumption equivalent, in percent. Capitalist welfare / capital value: welfare of the local capitalists in the island model and the value of local capital in the diversified model, in percent.

Figure G9: Comparative statics: capitalist risk aversion.

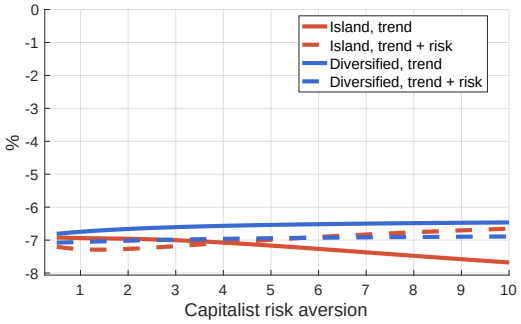
(a) Worker welfare, initial (% cons.)



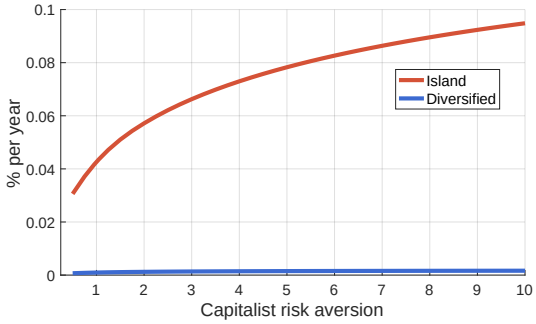
(b) Capitalist welfare, initial (% cons.)



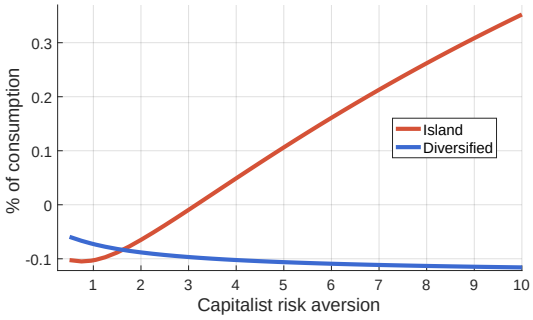
(c) Capital in 2100 (%)



(d) Capital risk premium, risk only (% per year)



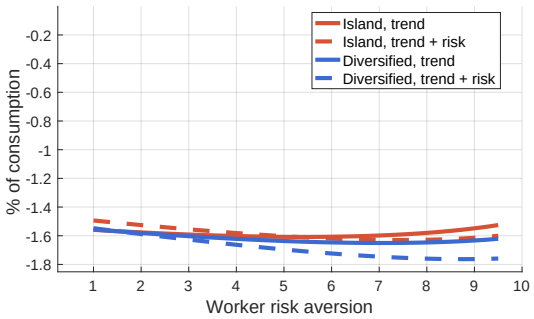
(e) Worker welfare, risk only (% cons.)



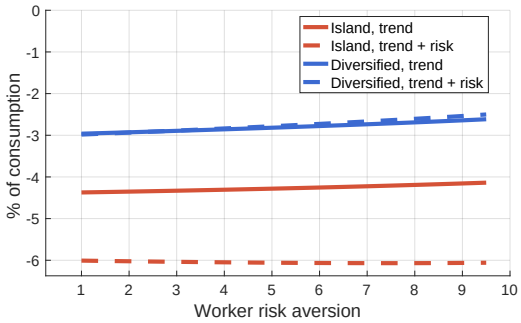
Note: Capitalist risk aversion $\overline{\gamma}$ from 0.5 to 10 at $\gamma = 2$, at 1,000 clusters. Panels (a) and (b): aggregate worker and capitalist welfare at the initial date of the 2°C transition, in percent of lifetime consumption, with the trend only (solid) and with the trend and all risk (dashed). Panel (c): change in the aggregate capital stock by 2100 relative to its initial level, in percent. Panels (d) and (e): capital risk premium, in percent per year, and aggregate worker welfare, in percent of lifetime consumption, in the stochastic steady state with the local and aggregate risk of a 2°C warmer climate. Island model in red, diversified model in blue.

Figure G10: Comparative statics: worker risk aversion.

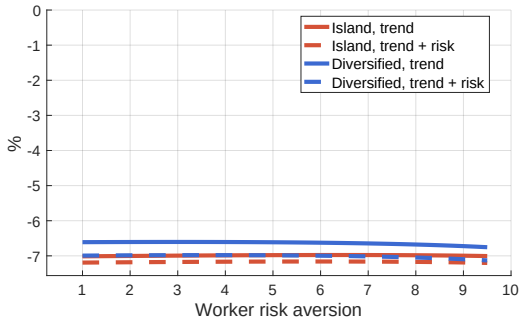
(a) Worker welfare, initial (% cons.)



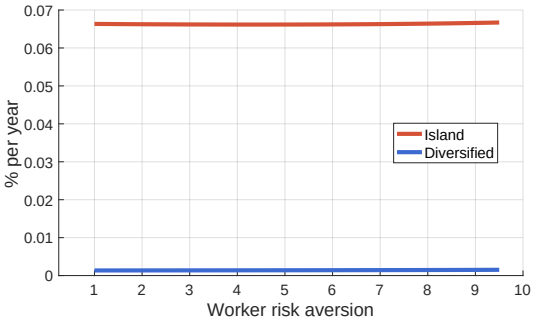
(b) Capitalist welfare, initial (% cons.)



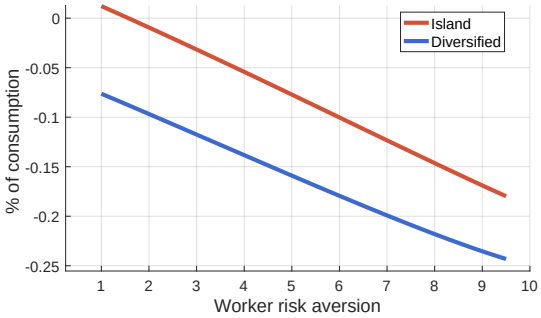
(c) Capital in 2100 (%)



(d) Capital risk premium, risk only (% per year)



(e) Worker welfare, risk only (% cons.)



Note: Worker risk aversion γ from 1 to 9.5 at $\overline{\gamma} = 3$, at 1,000 clusters. Panels and lines as in Figure G9.

H Extensions

This appendix develops two model extensions and their quantitative implications. Appendix H.1 extends the derivations for workers and capitalists to Epstein-Zin preferences. Appendix H.2 allows storm damages to vary across island capitalists within a county.

H.1 Epstein-Zin Preferences

We consider recursive preferences (Epstein and Zin, 1989) for workers and capitalists, using the continuous-time formulation of Duffie and Epstein (1992). We derive the general case with elasticity of intertemporal substitution (IES) φ for workers and $\overline{\varphi}$ for capitalists, but set both elasticities to one in the quantitative analysis.

H.1.1 Preferences, Steady State, and Inversion

We denote the inverse elasticities of intertemporal substitution by φ^{-1} and $\overline{\varphi}^{-1}$. We express the recursive aggregator in the same units as the CRRA value functions, leaving the migration elasticity ν, moving costs τ_{ij}, migration term $\mathcal{M}_i[V]$, and estimation targets unchanged. For a consumption index C and value V in these units, we define certainty-equivalent consumption as:

$$J(V) \equiv \left(1 + (1-\gamma)\,\rho V\right)^{\frac{1}{1-\gamma}}, \tag{H1}$$

which is the constant consumption level that delivers V under CRRA utility, since $u(J(V)) = \rho V$. The corresponding flow aggregator is:

$$\mathcal{F}(C, V) \equiv \frac{C^{1-\varphi^{-1}} J(V)^{\varphi^{-1}-\gamma} - J(V)^{1-\gamma}}{1-\varphi^{-1}} + \rho V. \tag{H2}$$

Replacing flow utility in the worker's problem (6) gives $\rho V_{it} = \mathcal{F}(\widetilde{C}_{it}, V_{it}) + \mathcal{M}_i[V] + \mathbb{E}_t[dV_{it}]/dt$. When $\varphi = 1/\gamma$, we recover the CRRA model, with $\mathcal{F}(C, V) = u(C)$. For $\varphi = 1$, $\mathcal{F}(C, V) = J(V)^{1-\gamma} \log\left(C/J(V)\right) + \rho V$. As in the baseline model, additive taste shocks enter through migration, so $\mathcal{M}_i[V]$ retains its log-sum-exp form. The main difference is that the flow payoff now depends on the worker's own value V_{it}, so the valuation of risk also depends on uncertain future consumption.

The aggregator has the following derivatives:

$$\mathcal{F}_C = C^{-\varphi^{-1}} J^{\varphi^{-1}-\gamma}, \quad \mathcal{F}_{CC} = -\varphi^{-1} C^{-\varphi^{-1}-1} J^{\varphi^{-1}-\gamma}, \quad \mathcal{F}_V = \rho \frac{\varphi^{-1}-\gamma}{1-\varphi^{-1}} \Big[\Big(\frac{C}{J} \Big)^{1-\varphi^{-1}} - 1 \Big],$$
$$\mathcal{F}_{CV} = \rho \, (\varphi^{-1} - \gamma) \, \frac{(C/J)^{1-\varphi^{-1}}}{C}, \qquad \mathcal{F}_{VV} = -\rho^2 (\varphi^{-1} - \gamma) \Big(\frac{C}{J} \Big)^{1-\varphi^{-1}} J^{\gamma-1}, \tag{H3}$$

where $J = J(V)$ and $dJ/dV = \rho J^\gamma$. For $\varphi = 1$, $\mathcal{F}_V = \rho(1-\gamma)\log(C/J)$. At steady state, $J(V_i^{ss})$ generally differs from $\widetilde{C}_i^{ss}$ because workers' values also reflect migration opportunities and taste shocks.

To recover amenities, we first obtain $U_i \equiv \rho V_i - \mathcal{M}_i[V]$ as in Appendix F. We then invert the steady-state worker equation $\rho V_i^{ss} = \mathcal{F}(\widetilde{C}_i^{ss}, V_i^{ss}) + \mathcal{M}_i[V^{ss}]$ to recover the consumption index:

$$\widetilde{C}_i = \Big[\big((1-\varphi^{-1})(U_i - \rho V_i) + J(V_i)^{1-\gamma} \big) J(V_i)^{\gamma-\varphi^{-1}} \Big]^{\frac{1}{1-\varphi^{-1}}},$$
$$\widetilde{C}_i = J(V_i) \exp\Big[\frac{U_i - \rho V_i}{J(V_i)^{1-\gamma}} \Big] \qquad \text{for } \varphi = 1.$$

We then recover $A_i = \widetilde{C}_i / \big(c_{0i}^w K_i^{(1-\omega-\varpi)\xi} N_i^{-(1-\varpi)\xi} \big)$. Since amenities are recovered up to a constant, we impose the same normalization as in the CRRA model, $\sum_i N_i \widetilde{C}_i^{1-\gamma} = 1$.

Capitalists have the same aggregator, replacing (φ, γ) with $(\overline{\varphi}, \overline{\gamma})$:

$$\overline{\mathcal{F}}(C, \mathcal{Q}) \equiv \frac{C^{1-\overline{\varphi}^{-1}} \overline{J}^{\overline{\varphi}^{-1}-\overline{\gamma}} - \overline{J}^{1-\overline{\gamma}}}{1-\overline{\varphi}^{-1}} + \rho \mathcal{Q}, \qquad \overline{J} \equiv \big(1 + (1-\overline{\gamma})\rho \mathcal{Q} \big)^{\frac{1}{1-\overline{\gamma}}}.$$

The capitalist steady state remains unchanged in both market structures. We invert the remaining fundamentals as in Appendix F.

H.1.2 Pricing and Master Equations

In the diversified economy, the stochastic discount factor follows Duffie and Epstein (1992):

$$\Lambda_t = \exp\Big\{ - \int_0^t [\rho - \overline{\mathcal{F}}_{\mathcal{Q}}(C_s, \mathcal{Q}_s)] \, ds \Big\} \, \overline{\mathcal{F}}_C(C_t, \mathcal{Q}_t).$$

Its dynamics are:

$$\begin{aligned}\frac{d\Lambda_t}{\Lambda_t} &= -\overline{R}_t\,dt - \left[\overline{\varphi}^{-1}\,\sigma^C_{z,t} + (\overline{\gamma}-\overline{\varphi}^{-1})\,\sigma^J_{z,t}\right]dW_t \\ &\quad -\sum_\ell \left[\overline{\varphi}^{-1}\,\sigma^C_{\chi,\ell,t} + (\overline{\gamma}-\overline{\varphi}^{-1})\,\sigma^J_{\chi,\ell,t}\right]dW_{\chi,\ell t} \\ &\quad -\sum_\ell \left[\overline{\varphi}^{-1}\,\sigma^C_{\Delta,\ell,t} + (\overline{\gamma}-\overline{\varphi}^{-1})\,\sigma^J_{\Delta,\ell,t}\right]dW_{\Delta,\ell t}, \end{aligned} \tag{H4}$$

where σ^C are the loadings of dC_t/C_t on the three families of shocks, as in (B21), and σ^J those of $d\log\overline{J}_t$. The stochastic discount factor weights consumption risk by the inverse IES and risk in certainty-equivalent values by the difference between risk aversion and the inverse IES. We use these loadings in the pricing derivation of Appendix B.2.

The risk-free rate of Proposition 1 then becomes:

$$\begin{aligned}\overline{R}_t &= \rho + \overline{\varphi}^{-1}\,\mu^C_t - \frac{\overline{\varphi}^{-1}(\overline{\varphi}^{-1}+1)}{2}\,\|\sigma^C_t\|^2 - \overline{\varphi}^{-1}(\overline{\gamma}-\overline{\varphi}^{-1})\,\sigma^C_t\cdot\sigma^J_t \\ &\quad + \frac{(\overline{\gamma}-\overline{\varphi}^{-1})(\overline{\varphi}^{-1}-1)}{2}\,\|\sigma^J_t\|^2. \end{aligned} \tag{H5}$$

The corresponding capital-pricing equation is:

$$\begin{aligned}\overline{R}_t Q_{it} = R_{it} &+ \frac{c_{i0}}{1+\zeta}Q_{it}^{1+\zeta} - \Delta_{it}Q_{it} + \mathbb{E}_t\left[\frac{dQ_{it}}{dt}\right] + \frac{1}{dt}\mathbb{C}\mathrm{ov}_t\Big(dQ_{it}, \frac{dK_{it}}{K_{it}}\Big) \\ &- \frac{1}{dt}\Big[\overline{\varphi}^{-1}\,\mathbb{C}\mathrm{ov}_t\Big(dQ_{it}, \frac{dC_t}{C_t}\Big) + (\overline{\gamma}-\overline{\varphi}^{-1})\,\mathbb{C}\mathrm{ov}_t\Big(dQ_{it}, \frac{d\overline{J}_t}{\overline{J}_t}\Big)\Big] \\ &- \frac{Q_{it}}{dt}\Big[\overline{\varphi}^{-1}\,\mathbb{C}\mathrm{ov}_t\Big(\frac{dK_{it}}{K_{it}}, \frac{dC_t}{C_t}\Big) + (\overline{\gamma}-\overline{\varphi}^{-1})\,\mathbb{C}\mathrm{ov}_t\Big(\frac{dK_{it}}{K_{it}}, \frac{d\overline{J}_t}{\overline{J}_t}\Big)\Big]. \end{aligned} \tag{H6}$$

To characterize the certainty-equivalent value, we introduce the additional state-space object $\mathcal{J}(z,\chi,N,\mathcal{K},t) \equiv \log\overline{J}_t$, whose Master Equation is:

$$\begin{aligned}\rho\,\mathcal{J} &= \overline{\mathcal{F}}^{\mathcal{J}}\left(\log\mathcal{C}(\chi,N,\mathcal{K},Q),\mathcal{J}\right) + \mathcal{L}_t\mathcal{J} - \frac{\overline{\gamma}-1}{2}\,\|\sigma^J_t\|^2, \\ \overline{\mathcal{F}}^{\mathcal{J}}(c,\mathcal{J}) &\equiv \frac{\rho}{1-\overline{\varphi}^{-1}}\Big[e^{(1-\overline{\varphi}^{-1})(c-\mathcal{J})}-1\Big] + \rho\mathcal{J}, \end{aligned} \tag{H7}$$

where $\mathcal{C}$ denotes aggregate capitalist consumption in the state space, $\mathcal{L}_t$ is the generator (15), and $\|\sigma^J_t\|^2$ is the instantaneous variance of $\mathcal{J}$, defined analogously to $\|\sigma^Q_{it}\|^2$ in Appendix C.2. For $\overline{\varphi}=1$, the flow reduces to $\rho\log\mathcal{C}$. For any IES, $\mathcal{J}^{ss} = \log C^{ss}$ at steady state.

Since $\mathcal{J}$ enters capital pricing only through its first-order loadings, the three Master Equations

form a triangular system. We first solve the FAME of (V, Q), followed by the FAME of $\mathcal{J}$. We then solve the SAME of (V, Q) using the known loadings σ^J, and finally the SAME of $\mathcal{J}$ to recover welfare.

For $\overline{\varphi}^{-1} \neq 1$, homogeneity gives the identity:

$$\overline{J}_t^{1-\overline{\varphi}^{-1}} = \rho\, C_t^{-\overline{\varphi}^{-1}}\, \Omega_t, \qquad \Omega_t \equiv \sum_i Q_{it} K_{it}, \tag{H8}$$

which extends the value-function identity of Appendix D.2. For $\overline{\varphi} = 1$, this reduces to the Merton rule $C_t = \rho\, \Omega_t$.

For island capitalists, the value-function guess (B25) remains valid, with certainty-equivalent consumption $\overline{J}_{it} = \rho^{1/(1-\overline{\gamma})} Q_{it} K_{it}$. The only modification to the Master Equation (C14) is its flow $\Omega_i(Q, R)$, which becomes:

$$\Omega_i(Q,R) = \max_g \left\{ \frac{\rho Q}{1-\overline{\varphi}^{-1}} \left[\left(\frac{R - c_i(g)}{\rho^{1/(1-\overline{\gamma})}\, Q} \right)^{1-\overline{\varphi}^{-1}} - 1 \right] + gQ \right\} + \rho Q. \tag{H9}$$

The steady-state derivatives of Ω_i and the investment policy $g_i(Q, R)$ follow from the CRRA expressions in Appendices C.3.2 and C.4.2, replacing $\overline{\gamma}$ with $\overline{\varphi}^{-1}$ and evaluating at the unchanged Q_i^{ss}. In particular:

$$\begin{aligned} g_{i,Q}^{ss} &= \frac{(1-\overline{\varphi}^{-1})\,\rho\zeta\Delta_i}{Q_i^{ss}(\rho + \overline{\varphi}^{-1}\zeta\Delta_i)}, \qquad g_{i,R}^{ss} = \frac{\overline{\varphi}^{-1}\,\zeta\Delta_i}{c_i'(\Delta_i)(\rho + \overline{\varphi}^{-1}\zeta\Delta_i)}, \\ \Omega_{i,QQ}^{ss} &= -\frac{\rho}{Q_i^{ss}} \frac{\overline{\varphi}^{-1}\rho + (2\overline{\varphi}^{-1} - 1)\zeta\Delta_i}{\rho + \overline{\varphi}^{-1}\zeta\Delta_i}. \end{aligned} \tag{H10}$$

The risk terms of (C14) retain $\overline{\gamma}$, since they arise from the nonlinear value function $(QK)^{1-\overline{\gamma}}$.

For $\overline{\varphi} = 1$, $g_{i,Q}^{ss} = 0$, and the investment first-order condition simplifies to $c_i'(g) = [R - c_i(g)]/\rho$. Investment therefore depends only on the rental rate, as in the Merton rule. As a result, risk affects island capitalists' welfare but not their investment decisions, and the no-covariance and general equilibrium exercises of Section 5.1 deliver the same capital and population allocations in the island model. The island aggregate-risk premium is also zero, since the rental rate does not load on the aggregate shock.

H.1.3 Expansions

In the worker Master Equation of Proposition 3, the flow payoff is $\mathcal{F}(\widetilde{C}_i, V_i)$. Its expansion has four parts: the expansion with respect to consumption $\widetilde{C}_i$, the expansion with respect to the worker's

value V_i, the continuation value from migration, and the continuation value from the evolution of the distribution and other shocks, captured by $\mathcal{L}_t$. Write the consumption index as $\widetilde{C}_{it} = \widetilde{C}_i^{ss} + d\widetilde{C}$, where $d\widetilde{C} = \epsilon\,\widetilde{C}_1 + \frac{\epsilon^2}{2}\widetilde{C}_2$ and $\widetilde{C}_1 = \widetilde{C}_i^{ss}\sum_x \mathrm{e}^{C}_{x,i}\,x_i$. To second order:

$$\begin{aligned}\rho\,dV &= \mathcal{F}_C\,d\widetilde{C} + \tfrac{1}{2}\mathcal{F}_{CC}\,d\widetilde{C}^2 + \mathcal{F}_V\,dV + \mathcal{F}_{CV}\,d\widetilde{C}\,dV + \tfrac{1}{2}\mathcal{F}_{VV}\,dV^2 \\ &\quad + d\mathcal{M}_i[V] + d\big(\mathcal{L}_t V_i\big),\end{aligned}$$

where all derivatives are evaluated at $(\widetilde{C}_i^{ss}, V_i^{ss})$. At first order, the flow is $\sum_x \mathrm{D}_{Ux}x + \mathrm{D}_{UV}v_1$, with D_{Ux} the matrix of Appendix C.3 evaluated with $\mathcal{F}_C$ in place of u', and:

$$\mathrm{D}_{UV} \equiv \mathbf{diag}\big(\mathcal{F}^{ss}_{V,i}\big), \qquad \mathcal{F}^{ss}_{V,i} = \rho\,\frac{\varphi^{-1}-\gamma}{1-\varphi^{-1}}\Big[\Big(\frac{\widetilde{C}_i^{ss}}{J(V_i^{ss})}\Big)^{1-\varphi^{-1}} - 1\Big], \tag{H11}$$

which equals $\rho(1-\gamma)\log\big(\widetilde{C}_i^{ss}/J(V_i^{ss})\big)$ for $\varphi = 1$. Since D_{UV} is diagonal, we absorb it into the operator, replacing M by $M + \mathrm{D}_{UV}$ wherever M multiplies the worker's own value. This shifts the effective discount rate in each worker row to $\rho - \mathcal{F}^{ss}_{V,i}$.

At second order, the consumption terms $\mathcal{F}_{CC}\widetilde{C}_1^2 + \mathcal{F}_C\widetilde{C}_2$ are collected by the tensors $\mathbb{D}_{Uxy}$ of Appendix C.4, with $\mathcal{F}_C$ and $\mathcal{F}_{CC}$ replacing u' and u''. The value terms add $\mathcal{F}_V v_2$, absorbed in the operator as at first order, and $2\mathcal{F}_{CV}\widetilde{C}_1 v_1$ and $\mathcal{F}_{VV}v_1^2$, whose tensors are given in Appendix H.1.5. The migration and generator terms are those of Appendix C.4.2, evaluated with the corresponding FAME and SAME objects.

For island capitalists, the aggregator's dependence on their own value is already captured by Q_{it} in the flow $\Omega_i(Q,R)$ of (H9), so no additional term arises. For the diversified capitalist, the pricing equation uses $\overline{\varphi}^{-1}$ instead of $\overline{\gamma}$ in $\mathcal{R}_{it}$, while the $(\overline{\gamma} - \overline{\varphi}^{-1})\sigma^J$ terms in the prices of risk are second order and enter only ι^Q_{it}.

We approximate the Master Equation (H7) of $\mathcal{J}$ to first and second order, as for V and Q. We expand it as $\mathcal{J} = \mathcal{J}^{ss} + \epsilon\,\jmath_{1t} + \frac{\epsilon^2}{2}\jmath_{2t}$, with $\jmath_{1t} = \jmath^n n + \jmath^k k + \jmath^\chi\widehat{\chi} + \jmath^z_t z + \jmath^T_t$, where $\jmath^n, \jmath^k, \jmath^\chi$ are $1\times\mathcal{I}$ rows and $\jmath^z_t, \jmath^T_t$ are scalars. To second order:

$$\jmath_{2t} = \sum_{x,y} x^*\jmath^{x,y}\,y + 2\sum_x \jmath^{x,z}_t\,x\,z + 2\sum_x \jmath^{x,T}_t\,x + \jmath^{zz}_t\,z^2 + \jmath^{TT}_t + 2\,\jmath^{zT}_t\,z + \jmath^{\Sigma,z}_t\,z + \jmath^{\Sigma,T}_t + \bar{\jmath}_t,$$

which follows the form of (16), with $\mathcal{I}\times\mathcal{I}$ matrices $\jmath^{x,y}$, vectors $\jmath^{x,z}_t$ and $\jmath^{x,T}_t$, and scalar coefficients for the remaining terms. To second order, the flow of (H7) is $\rho\,d\log\mathcal{C} + \frac{1}{2}\rho(1-\overline{\varphi}^{-1})(d\log\mathcal{C} - d\mathcal{J})^2$.

H.1.4 FAME

The FAME of Appendix C.3.2 applies with three modifications. For workers, D_{Ux} is evaluated with $\mathcal{F}_C(\widetilde{C}_i^{ss}, V_i^{ss})$ in place of $u'(\widetilde{C}_i^{ss})$, and the flow becomes $\sum_x \mathrm{D}_{Ux} x + \mathrm{D}_{UV} v_{1t}$. We also replace M with $M + \mathrm{D}_{UV}$ in the worker rows of the FAME, including in $\overline{\Psi}$. The migration response G, the population law, the lemmas of Appendix C.1, and the form of Θ are unchanged. For island capitalists, we use $g_{i,Q}^{ss}$ and $g_{i,R}^{ss}$ from (H10), replacing $\overline{\gamma}$ with $\overline{\varphi}^{-1}$ in $\mathrm{D}_{\mathcal{I}Q}$, $\mathrm{D}_{\mathcal{I}R}$, and $\mathrm{D}_{\mathcal{I}x}$. The flow matrices are unchanged. For the diversified capitalist, $\overline{\varphi}^{-1}$ replaces $\overline{\gamma}$ in the six interest rate matrices of (C20), giving $\overline{R}_t = \rho + \overline{\varphi}^{-1}\,\mu_t^C$ to first order. The diversified $\mathrm{D}_{\mathcal{I}Q}$ does not depend on $\overline{\gamma}$.

The diversified model has one additional first-order object, the certainty-equivalent value $\mathcal{J}$. For any IES, its first-order flow is $\rho\, C_{1t}/C^{ss}$, where $C_{1t} = \sum_x \mathcal{C}_x x + \mathcal{C}_Q q_{1t}$ is the first-order deviation of aggregate consumption. Defining the total consumption loadings $\overline{\mathcal{C}}^x \equiv \mathcal{C}_x + \mathcal{C}_Q Q^x$, we obtain:

$$\begin{aligned} \left[\mathcal{J}^n, \mathcal{J}^k, \mathcal{J}^\chi\right](\rho\,\mathbf{I} - \Theta) &= \frac{\rho}{C^{ss}}\left[\overline{\mathcal{C}}^n, \overline{\mathcal{C}}^k, \overline{\mathcal{C}}^\chi\right], \\ (\rho+\theta)\,\mathcal{J}_t^z - \partial_t \mathcal{J}_t^z &= \frac{\rho}{C^{ss}}\,\mathcal{C}_Q q_t^z + \sum_s \mathcal{J}^s\,\Theta_t^{s,z}, \qquad \rho\,\mathcal{J}_t^T - \partial_t \mathcal{J}_t^T = \frac{\rho}{C^{ss}}\,\mathcal{C}_Q q_t^T + \sum_s \mathcal{J}^s\,\Theta_t^{s,T}. \end{aligned} \tag{H12}$$

The loadings of $\log \overline{J}_t$ on the shocks are the counterparts of (C2): $\sigma_{z,t}^J = \epsilon \mathcal{J}_t^z$, $\sigma_{\chi,\ell,t}^J = \Sigma_{\chi,\ell t}\mathcal{J}_\ell^\chi$, and $\sigma_{\Delta,\ell,t}^J = -\Sigma_{\Delta,\ell t}\mathcal{J}_\ell^k$.

H.1.5 SAME

For workers in both models, the tensors $\mathbb{D}_{Uxy}$ of Appendix C.4 are evaluated with $\mathcal{F}_C$ and $\mathcal{F}_{CC}$ replacing u' and u'' throughout. The aggregator also adds the value terms $2\mathcal{F}_{CV}\widetilde{C}_1 v_1$ and $\mathcal{F}_{VV} v_1^2$ of Appendix H.1.3. Define:

$$\mathrm{D}_{UVV} \equiv \mathbf{diag}\big(\mathcal{F}_{VV,i}^{ss}\big), \qquad \mathbb{D}_{UVx} \equiv \mathbf{diag}_3\big(\mathcal{F}_{CV,i}^{ss}\,\widetilde{C}_i^{ss}\,\mathrm{e}_{x,i}^C\big), \tag{H13}$$

where the derivatives in (H3) are evaluated at steady state. We expand $\frac{1}{2}\mathcal{F}_{VV}(\epsilon v_1)^2$ using Lemma 2 and the outer products $\mathbb{V}^{x,y}$.

The worker forcings of Appendix C.4.2 change as follows. The tensor $\mathbb{V}_0^{x,y}$ gains $\mathbb{V}^{x,y} \times_1 \mathrm{D}_{UVV} + \mathbb{D}_{UVx} \times_3 V^y + \mathbb{D}_{UVy} \times_2 V^x$. The forcing $\mathbf{V}_{0t}^{x,z}$ gains $\mathrm{D}_{UVV}\mathbf{diag}(v_t^z)V^x + \mathbb{D}_{UVx} \times_3 v_t^z$, and the xT block gains the same terms with v_t^T in place of v_t^z. Finally, $\mathbf{v}_{0t}^{z,z}$, $\mathbf{v}_{0t}^{T,T}$, and $\mathbf{v}_{0t}^{z,T}$ gain $\mathrm{D}_{UVV}(v_t^z \cdot v_t^z)$, $\mathrm{D}_{UVV}(v_t^T \cdot v_t^T)$, and $\mathrm{D}_{UVV}(v_t^z \cdot v_t^T)$, respectively. The worker Σ and constant forcings are unchanged, and no variance correction is needed because the worker value is itself the utility in-

dex. In the operator Φ of (C56), the worker block $M + \mathrm{X}_V$ becomes $M + \mathrm{D}_{UV} + \mathrm{X}_V$. The same replacement applies in the worker rows of the cross, stochastic, and constant blocks.

For island capitalists, the second derivatives of Ω_i and g_i in (C60) and (C61) use $\overline{\varphi}^{-1}$ instead of $\overline{\gamma}$, as do the flow and capital-law tensors of Appendix C.4 built from them. The risk forcings $\mathrm{d}_{Q\Sigma T}$, $\mathrm{d}_{Q\Sigma K}$, $\mathrm{d}_{Q\Sigma Q}$, and $\mathrm{d}_{Q\Sigma z,t}$ retain $\overline{\gamma}$, since they come from the value function $(QK)^{1-\overline{\gamma}}$. No additional object is needed.

For the diversified capitalist, $\overline{\varphi}^{-1}$ replaces $\overline{\gamma}$ in the risk-free rate and the Itô correction of Appendix C.4. This applies to the ϱ rows, the families $\mathbb{J}$, $\mathbb{K}$, and $\mathbb{L}$, the local covariance matrices of (C50), and the forcing (C54). The terms involving $\overline{\gamma} - \overline{\varphi}^{-1}$ in (H5) and (H6) add a correction to this forcing. Let $\iota_t^Q|_{\overline{\varphi}^{-1}}$ denote (C54) with $\overline{\varphi}^{-1}$ in place of $\overline{\gamma}$. Using the loadings $\jmath^\chi$, $\jmath^k$, and $\jmath_t^z$ of (H12) and the consumption loadings $\overline{c}^\chi$, $\overline{c}^k$, and $\mathcal{C}_Q q_t^z$ of Appendix C.4, we obtain:

$$\begin{aligned}
\iota_t^Q - \iota_t^Q|_{\overline{\varphi}^{-1}} = \frac{\epsilon^2}{2}(\overline{\gamma} - \overline{\varphi}^{-1})\Big\{ & -2\Big[Q^\chi \Sigma_\chi (\jmath^\chi)^* + Q^k \Sigma_\Delta (\jmath^k)^*\Big](\sigma_t z + \varsigma_t) \\
& - 2\,\mathbf{diag}(Q^{ss})\,\Sigma_\Delta(\jmath^k)^*(\sigma_t z + \varsigma_t) - 2\,\jmath_t^z q_t^z \\
& + Q^{ss}\Big[2\overline{\varphi}^{-1}\,\frac{\overline{c}^\chi \Sigma_\chi (\jmath^\chi)^* + \overline{c}^k \Sigma_\Delta (\jmath^k)^*}{C^{ss}} \\
& \qquad - (\overline{\varphi}^{-1} - 1)\Big(\jmath^\chi \Sigma_\chi (\jmath^\chi)^* + \jmath^k \Sigma_\Delta (\jmath^k)^*\Big)\Big](\sigma_t z + \varsigma_t) \\
& + Q^{ss}\Big[2\overline{\varphi}^{-1}\,\frac{(\mathcal{C}_Q q_t^z)\,\jmath_t^z}{C^{ss}} - (\overline{\varphi}^{-1} - 1)(\jmath_t^z)^2\Big]\Big\}.
\end{aligned} \tag{H14}$$

The first two lines contain the price-of-risk terms $-(\overline{\gamma} - \overline{\varphi}^{-1})\sigma^J \cdot \sigma^Q$ and, in the term with $\mathbf{diag}(Q^{ss})$, the quantity channel $(\overline{\gamma} - \overline{\varphi}^{-1})\, Q_i \Sigma_{\Delta,i} \sum_\ell \mathfrak{c}_{\Delta,i\ell}\sigma^J_{\Delta,\ell}$. The remaining three lines capture the two additional terms in the risk-free rate (H5), multiplied by $-Q_i$. As in Appendix C.4.2, the $\sigma_t z$ terms enter $\mathbf{q}_{0t}^{\Sigma,z}$, the ς_t terms enter $\mathbf{q}_{0t}^{\Sigma,T}$, and the $\jmath_t^z$ terms enter $\overline{\mathbf{q}}_{0t}$. The no-covariance valuation of Appendix D.3 drops the local-risk price-of-risk terms along with $\mathrm{D}_{Q\Sigma\chi}$, $\mathrm{D}_{Q\Sigma\Delta}$, and $\mathrm{d}_{Q\Sigma K}$, but retains the aggregate term $\jmath_t^z q_t^z$ of the constant block, the quantity channel, and the risk-free-rate terms.

For the certainty-equivalent value $\mathcal{J}$, we expand (H7) to second order, following Appendix C.4. The flow consists of the second-order expansion of $\rho \log \mathcal{C}$ and the aggregator curvature $\frac{1}{2}\rho(1 - \overline{\varphi}^{-1})[(\log \mathcal{C})_1 - \jmath_{1t}]^2$ when $\overline{\varphi}^{-1} \neq 1$. The second-order drifts of the population and capital distributions are weighted by the rows $\jmath^n$ and $\jmath^k$. The term $-\frac{\overline{\gamma}-1}{2}\|\sigma_t^J\|^2$ enters the Itô correction. The (x, y) blocks, stacked in the $3\mathcal{I} \times 3\mathcal{I}$ matrix $\jmath \equiv [\jmath^{x,y}]_{x,y}$, solve:

$$\rho\,\jmath = \mathbb{F}^{\mathcal{J}} + \mathcal{T} \times_1 \left(u^{\mathcal{J}}\right)^* + \Theta^* \jmath + \jmath\,\Theta, \qquad u^{\mathcal{J}} \equiv \begin{pmatrix} G^*(\jmath^n)^* \\ \mathrm{D}^*_{\mathcal{I}Q}(\jmath^k)^* + \rho\,\mathcal{C}^*_Q / C^{ss} \end{pmatrix}, \tag{H15}$$

where $\mathcal{T}$ is the solved tensor of (C56), and $u^{\mathcal{J}}$ collects the weights on v_2 and q_2 arising from $\rho(\log \mathcal{C})_2$, $\jmath^n G v_2$, and $\jmath^k \mathrm{D}_{\mathcal{I}Q} q_2$. The forcing $\mathbb{F}^{\mathcal{J}}$ collects the remaining known terms: the second-order expansion of $\rho \log \mathcal{C}$ net of q_2, the migration and capital terms of Appendix C.4.2 weighted by $\jmath^n$ and $\jmath^k$, and the aggregator curvature. This is a matrix Sylvester equation of the form (C56).

The cross, stochastic, and constant blocks of $\jmath_{2t}$ are obtained similarly, using the corresponding blocks of (V, Q) weighted by $u^{\mathcal{J}}$, along with their own terms. The curvature $-(\overline{\gamma}-1)[\jmath^k \Sigma_\Delta (\jmath^k)^* + \jmath^\chi \Sigma_\chi (\jmath^\chi)^*]$ enters the Itô terms of the Σ blocks, and $-(\overline{\gamma}-1)(\jmath^z_t)^2$ enters the constant block. The paths of $\jmath_{1t}$ and $\jmath_{2t}$ determine the welfare of the diversified capitalist in (H20).

H.1.6 Risk Premia and Welfare

For the diversified capitalist, the risk-free rate is given by (H5). The risk premium follows from (H6), replacing the CRRA prices of risk $\overline{\gamma}\,\sigma^C$ with $\overline{\varphi}^{-1}\,\sigma^C + (\overline{\gamma} - \overline{\varphi}^{-1})\,\sigma^J$ from (H4). For island capitalists, the stochastic discount factor takes the same form as (H4), using own consumption C^{cap}_{it} and certainty-equivalent consumption $\overline{J}_{it}$. Since $\overline{J}_{it}$ is proportional to $Q_{it}K_{it}$, the loadings σ^J_{it} are those of $\log(Q_{it}K_{it})$. The shadow risk-free rate and premium become:

$$\begin{aligned} R^f_{it} &= \rho + \overline{\varphi}^{-1}\,\mu^C_{it} - \frac{\overline{\varphi}^{-1}(\overline{\varphi}^{-1}+1)}{2}\|\sigma^C_{it}\|^2 - \overline{\varphi}^{-1}(\overline{\gamma}-\overline{\varphi}^{-1})\,\sigma^C_{it}\cdot\sigma^J_{it} \\ &\quad + \frac{(\overline{\gamma}-\overline{\varphi}^{-1})(\overline{\varphi}^{-1}-1)}{2}\|\sigma^J_{it}\|^2, \qquad \text{(H16)} \\ \pi_{it} &= \frac{1}{dt}\mathbb{E}_t\left[dR^K_{it}\right] - R^f_{it} = \left[\overline{\varphi}^{-1}\,\sigma^C_{it} + (\overline{\gamma}-\overline{\varphi}^{-1})\,\sigma^J_{it}\right]\cdot\sigma^{R^K}_{it}, \qquad \text{(H17)} \end{aligned}$$

where $\sigma^{R^K}_{it}$ collects the loadings of the return on installed capital, including own-capital destruction $-\Sigma_{\Delta,it}$ and the loadings of the price q_{it}. Both C^{cap}_{it} and $\overline{J}_{it}$ fall one for one with the destruction of own capital, so the quantity channel of the premium remains $\overline{\gamma}\,\Sigma_{\Delta,iit}$ regardless of the IES, as in (D4). The remaining terms capture the consumption and price hedges of Appendix D.1, with $\overline{\varphi}^{-1}\,\partial \log \bar{c}_{it} + (\overline{\gamma}-\overline{\varphi}^{-1})\,\partial \log Q_{it}$ replacing $\overline{\gamma}\,\partial \log \bar{c}_{it}$, where $\bar{c}_{it}$ is consumption per unit of capital.

The worker consumption equivalent ce_{is} is defined as in Appendix D.2. Let $V_i(\mathrm{ce})$ denote the value of the consumption path $(1+\mathrm{ce})\widetilde{C}^{ss}$, holding the migration policy fixed at its steady-state value. It satisfies the steady-state worker equation of Appendix H.1.1, with $(1+\mathrm{ce})\widetilde{C}^{ss}_j$ in place of $\widetilde{C}^{ss}_j$, and $V_i(0) = V^{ss}_i$. The migration term is linear in $V(\mathrm{ce})$, with matrix M from Appendix C.3. However, because $\mathcal{F}$ is nonlinear in V, $V_i(\mathrm{ce})$ has no closed-form solution. We therefore expand it to second order around $\mathrm{ce} = 0$. Writing $W_i \equiv V_i'(0)$ and $W_{2,i} \equiv V_i''(0)$, the vectors W and W_2

solve:

$$\begin{aligned}\left[\rho\,\mathbf{I} - \mathrm{D}_{UV} - M\right] W &= \mathcal{F}_C \cdot \widetilde{C}^{ss}, \\ \left[\rho\,\mathbf{I} - \mathrm{D}_{UV} - M\right] W_2 &= \mathcal{F}_{CC} \cdot (\widetilde{C}^{ss})^2 + 2\,\mathcal{F}_{CV} \cdot \widetilde{C}^{ss} \cdot W + \mathcal{F}_{VV} \cdot W^2,\end{aligned} \tag{H18}$$

where vector products are entrywise and the derivatives of $\mathcal{F}$ are evaluated at each location's steady state. Solving $V_i(\mathrm{ce}_{is}) = V_i^{ss} + dV_{is}$ to second order gives:

$$\mathrm{ce}_{is} = \mathrm{ce}_{is}^{(1)} - \frac{1}{2}\,\frac{W_{2,i}}{W_i}\left(\mathrm{ce}_{is}^{(1)}\right)^2, \qquad \mathrm{ce}_{is}^{(1)} \equiv \frac{dV_{is}}{W_i}. \tag{H19}$$

We aggregate worker consumption equivalents weighting locations by steady-state population and real income, as in Appendix D.2.

For island capitalists, the consumption-equivalent formula is unchanged. Since preferences are homothetic and certainty-equivalent consumption $\overline{J}_{is}$ is proportional to $Q_{is}K_{is}$ for any IES, $\mathrm{ce}_{is}^{cap} = \overline{J}_{is}/\overline{J}_i^{ss} - 1$ coincides with (D7). For the diversified capitalist, we compute $\overline{J}_s = \exp(\mathcal{J}_s)$ using the FAME and SAME of Appendices H.1.4 and H.1.5. The consumption equivalent is:

$$\mathrm{ce}_s^{cap} = \exp\left(\epsilon\,\jmath_{1s} + \frac{\epsilon^2}{2}\,\jmath_{2s} + \log\frac{C^{ss,f}}{C^{ss,i}}\right) - 1, \tag{H20}$$

where $\jmath_{1s}$ and $\jmath_{2s}$ are the first- and second-order terms in the expansion of $\mathcal{J}$ around the final steady state (in the transition, around the steady state used at date s, whose consumption then replaces $C^{ss,f}$, with $\mathcal{J}$ pasted and averaged across expansion points like values and prices). The last term adjusts for the difference between final and initial steady-state consumption, $C^{ss,f}$ and $C^{ss,i}$, so welfare is measured relative to the initial steady state. It vanishes when the two steady states coincide.

H.1.7 Results

We set the elasticities of intertemporal substitution for workers and capitalists to one, $\varphi = \overline{\varphi} = 1$, and retain the baseline risk aversions, $\gamma = 2$ and $\overline{\gamma} = 3$. We then re-estimate the model's elasticities and climate damages. The estimates for the investment elasticity and the storm and heat shocks are essentially unchanged. The migration elasticity rises slightly from 0.92 to 1.0. Figures H1 and H2 repeat the exercises of Figures 7, 8, and 9, using the new estimates.

Local risk continues to dominate aggregate risk. Island capitalists require premia of up to 3.1 p.p. per year to hold capital exposed to local risk, somewhat above the 2.7 p.p. under CRRA.

In contrast, the island aggregate-risk premium is exactly zero with a unit IES, as implied by the Merton rule, compared with an already negligible 0.006 p.p. under CRRA. Over a lifetime, the most exposed island capitalists lose 18% of consumption from local risk and 11% from aggregate risk, close to the respective losses of 19% and 14% under CRRA.

Figure H1 shows that price responses and diversification continue to insure investors. For coastal island capitalists, the risk premium is 1.5 p.p. per year without the price covariance and 0.7 with it, essentially as under CRRA. Their general equilibrium loss in lifetime consumption is 2.6%, close to 3.1% under CRRA. Without the price covariance, however, the loss is 12%, far below 24% under CRRA. For the diversified capitalist, welfare remains at its CRRA values, and the value of capital in Figure H1(b) is essentially unchanged.

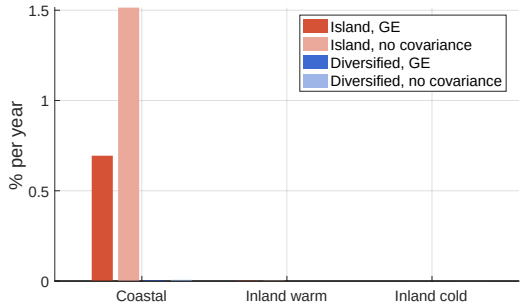
The main difference from CRRA is the smaller no-covariance welfare loss, which reflects the absence of precautionary investment. Under CRRA, island capitalists accumulate capital in response to aggregate risk, with increases of up to 5.3% in a county. They also build buffer stocks when the price covariance is removed. With a unit IES, however, the investment rate no longer responds to the value of capital, so precautionary accumulation almost disappears: capital rises by at most 0.4% in any county. As a result, the no-covariance and general equilibrium economies have the same capital and population allocations in the island model. The no-covariance welfare loss therefore reflects only the absence of the price hedge.

Workers are barely affected by risk in either model, with coastal losses of about 0.1% of lifetime consumption. The main difference from CRRA appears when the price covariance is removed. Under CRRA, workers gain because island capitalists build buffer stocks in capital. With a unit IES, no such buffer stocks are built. Capital and wages are therefore the same with and without the price covariance, so the light and dark red bars of Figure H1(c) coincide. Separating risk aversion from intertemporal substitution thus reduces the worker gains from precautionary accumulation. These differences arise from the effect of the IES on investment rather than a change in risk aversion. In particular, the quantity channel of the risk premium, $\overline{\gamma}\Sigma_{\Delta,iit}$, is independent of the IES.

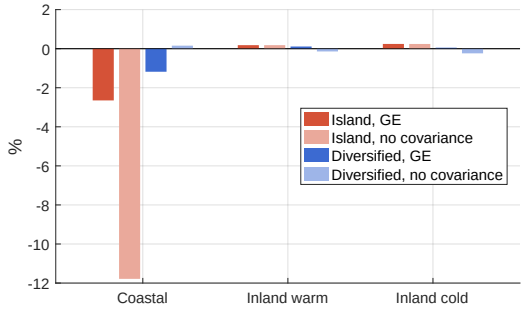
Finally, Figure H2 shows that risk remains less costly than the trend. By 2100, the capital stock falls by 7.2% in the island model and 7.0% in the diversified model, virtually the same numbers as under CRRA. Unlike under CRRA, island capital does not rise in the first years, since the investment rate no longer responds to the value of capital. Island capitalists' welfare in 2100 is -4.0% with the trend alone and -4.8% with the trend and all risk. For the diversified capitalist, the corresponding values are -3.8% and -4.0%, all very close to the CRRA results.

Figure H1: Capital risk premium and welfare cost of climate risk with and without the price covariance, by location group (Epstein-Zin preferences).

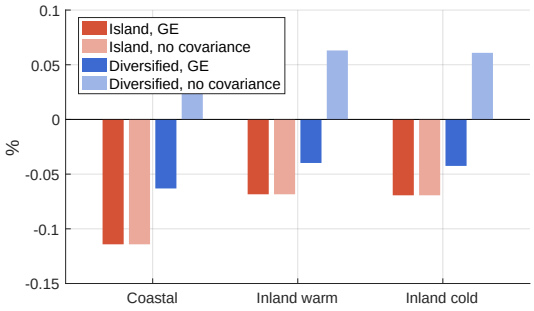
(a) Capital risk premium (% per year)



(b) Capitalist welfare / capital value (%)



(c) Worker welfare (%)



Note: Epstein-Zin preferences with a unit elasticity of intertemporal substitution, risk aversions $\gamma = 2$ and $\overline{\gamma} = 3$, and re-estimated elasticities and damages, at the county level (3,037 counties). Effect of all risk in 2100 along the 2°C transition. Panel (a): capital risk premium, in percent per year, capital-weighted within each group. Panels (b) and (c): welfare with the trend and all risk minus welfare with the trend only, in percent of lifetime consumption. Panel (b) reports the welfare of island capitalists in the island model and the value of local capital in the diversified model. Panel (c): worker welfare, weighted by consumption and population. Island model in red, diversified model in blue. Dark bars: general equilibrium. Light bars ('no covariance'): the premium in panel (a) and the valuation in panels (b) and (c) without the covariance between the price of capital and local shocks (Section 5.1). Groups as in Figure 7.

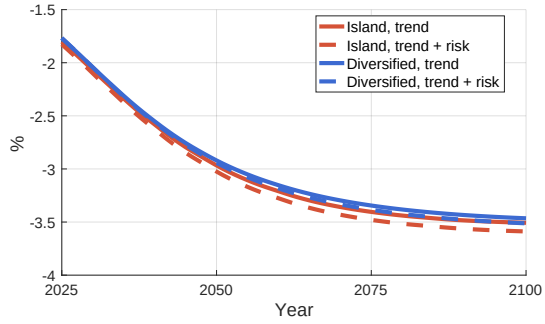
H.2 Within-County Dispersion of Storm Damages

H.2.1 Microfoundation

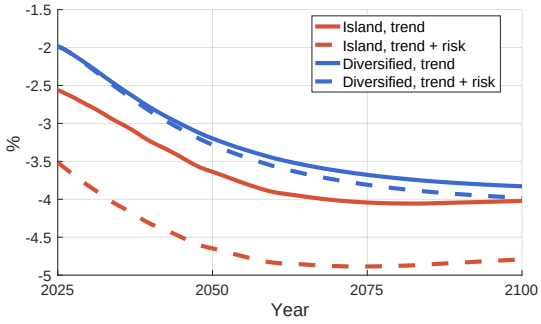
In our baseline framework, a storm destroys a fraction D_i of all capital in the location it hits. We now consider an alternative in which a continuum of owners $j \in [0,1]$ holds the capital stock in each location, and damages differ across owners. Shocks arrive as before, but when one hits i, owner j loses the fraction $D_i d_{ij}$ of its capital. The multipliers $d_{ij} \geq 0$ are drawn independently for each shock, owner, and location. They have mean $\mathbb{E}[d_{ij}] = 1$ and variance $\mathbb{V}\text{ar}(d_{ij}) = \Sigma_0^2$, common across locations. The capital of owner j follows $dK_{ijt}/K_{ijt} = g_{it}\,dt - d\overline{\Delta}_{ijt}$, with $d\overline{\Delta}_{ijt} = \Delta_i\,dt + D_i\,d_{ijt}\,dJ_{it}$. Because the multipliers are independent of capital holdings when a storm hits,

Figure H2: Aggregate effects of warming with and without risk, 2025 to 2100 (Epstein-Zin preferences).

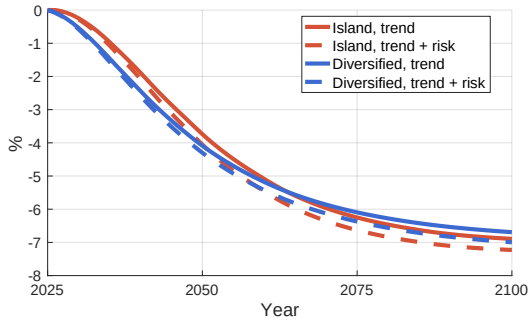
(a) Worker welfare



(b) Capitalist welfare



(c) Capital



Note: Epstein-Zin preferences as in Figure H1. Transition to a 2°C warmer climate by 2100. Island model in red, diversified model in blue. Solid lines: trend only. Dashed lines: trend with local and aggregate risk. Panel (a): aggregate worker welfare, the lifetime consumption equivalent at each date weighted by consumption and population, in percent. Panel (b): aggregate capitalist welfare, the consumption-weighted lifetime consumption equivalent of island capitalists and the diversified capitalist's equivalent, in percent. Panel (c): aggregate capital stock relative to 2025, in percent.

the law of large numbers gives $\int_0^1 d_{ijt} K_{ijt}\, dj = K_{it}$. Thus, the location still loses the fraction D_i of its capital, as in the baseline model. Dispersion leaves aggregate storm damage unchanged but increases the risk faced by each owner. Setting $\Sigma_0 = 0$ recovers the baseline of Appendix A.1.

To characterize this risk, we use the moments of the damage multipliers. Independence of d_{ij} from storm arrivals implies $\mathbb{E}[d_{ij}\, dJ_{it}] = p_{it}\, dt$, $\mathbb{E}[d_{ij}^2\, dJ_{it}] = (1+\Sigma_0^2)\, p_{it}\, dt$, and $\mathbb{E}[d_{ij}\, dJ_{it}\, dJ_{\ell t}] = p_{i\ell,t}\, dt$ for $\ell \neq i$. Expanding the owner's jump generator to second order leaves the location-level terms of (A1) unchanged. The expected destruction rate remains $D_i p_{it}$, as for the location, but the variance rises to $D_i^2(1+\Sigma_0^2)p_{it}$, an increase of $D_i^2\Sigma_0^2 p_{it}$. Covariances with the destruction rates of the owner's location and other locations are unchanged, while the destruction shocks of two owners in the same location have correlation $1/(1+\Sigma_0^2)$.

In the diffusion limit, we retain the location innovations $dW_{\Delta,it}$ of Appendix A.1 and add one

innovation dW_{ijt} per owner. These are independent standard Brownian motions, also independent of all location innovations. We retain the damage fractions D_i and the arrival rates of Appendix A.1. As with the constant part of the variance, we hold Σ_0 fixed and of order one as $\epsilon \to 0$, so that the additional risk remains a second-order term. Defining the effective loading $\widetilde{\Sigma}_{\Delta,i} \equiv \Sigma_{\Delta,i}\sqrt{1+\Sigma_0^2}$ and volatility $\widetilde{\Sigma}_{\Delta,it} \equiv \Sigma_{\Delta,it}\sqrt{1+\Sigma_0^2}$, we obtain:

$$\begin{aligned} d\overline{\Delta}_{it} &= \Delta_{it}\,dt + \Sigma_{\Delta,it}\,dW_{\Delta,it}, \qquad \Sigma_{\Delta,it} = \epsilon\,\Sigma_{\Delta,i}\sqrt{\vartheta_{\Delta,i} + \sigma_t z_t + \varsigma_t}, \\ d\overline{\Delta}_{ijt} &= \Delta_{it}\,dt + \Sigma_{\Delta,it}\big(dW_{\Delta,it} + \Sigma_0\,dW_{ijt}\big). \end{aligned} \tag{H21}$$

The variance of the owner's destruction rate is $\mathbb{V}\mathrm{ar}_t(d\overline{\Delta}_{ijt})/dt = (1+\Sigma_0^2)\,\Sigma_{\Delta,it}^2 = \widetilde{\Sigma}_{\Delta,it}^2$: dispersion increases the constant and temperature components of each owner's variance by the same factor. Covariances across locations remain unchanged because the owner-specific innovations are independent and average out within each location.

The location-level state and the covariance matrix of its innovations are unchanged, so the FAME is also unchanged. The diversified model also retains its original form, since the capitalist holds the capital of every owner across all locations. In the island model, however, each capitalist prices and consumes from its own capital, so the additional risk enters the second-order solution through the variance of own capital destruction. In the Bellman equation (11), the own second-order term uses the variance $(1+\Sigma_0^2)\Sigma_{\Delta,it}^2$ instead of $\Sigma_{\Delta,it}^2$, while the cross terms with location-level capital retain the original covariances. Consequently, the certainty-equivalent depreciation rate in (C14) becomes $\Delta_{it} + \overline{\gamma}(1+\Sigma_0^2)\Sigma_{\Delta,it}^2/2$. This affects the island investment rate, capital path, and certainty-equivalent wealth $Q_{it}K_{it}$. The quantity channel of the premium (D4) becomes $\overline{\gamma}(1+\Sigma_0^2)\Sigma_{\Delta,iit}$, and the own term of the consumption variance in the shadow risk-free rate (D3) is scaled in the same way. The two hedges still depend on $\Sigma_{\Delta,i}$ because rental rates and capital prices respond to aggregate damage in the location, not the additional losses of individual owners.

County-level data identify aggregate storm damage within a location but not its distribution across owners. Identifying Σ_0 would require information on damages across establishments or properties affected by the same storm, which county-level storm counts do not provide. We therefore report a sensitivity analysis in Σ_0 for the island model.

H.2.2 Results

We repeat the geography exercise of Section 5.1 in the island model, considering local and aggregate risk in a 2°C warmer climate and varying Σ_0 from 0 to 1. The diversified model remains as in

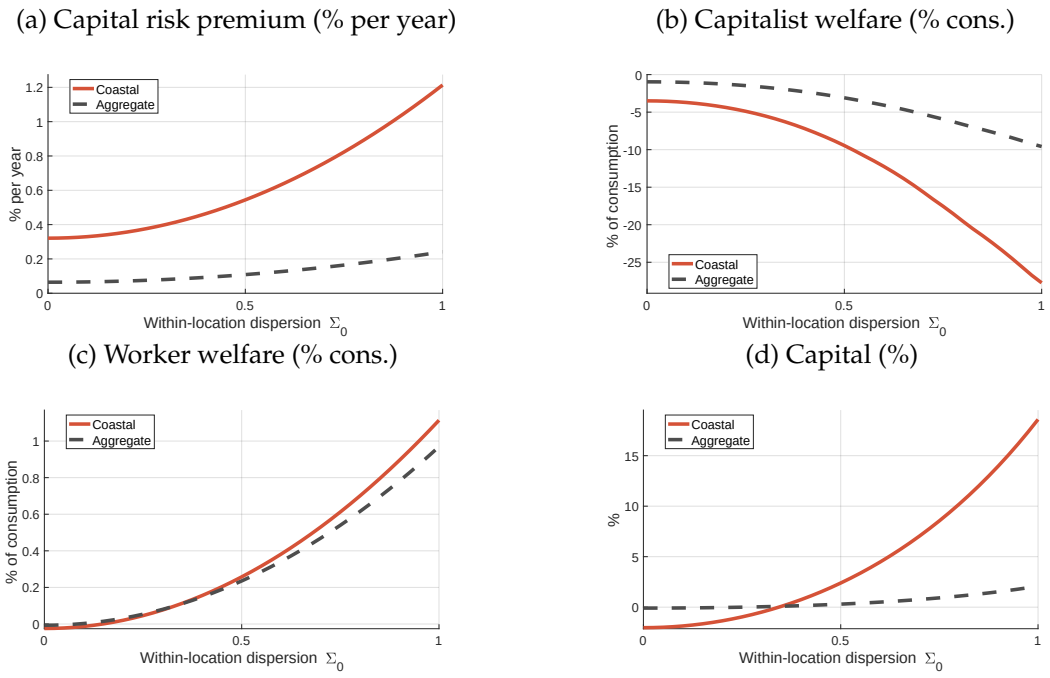
the baseline, since dispersion leaves the aggregate county losses borne by the national capitalist unchanged. For example, $\Sigma_0 = 1$ corresponds to a two-point distribution in which half of the owners in a county lose twice the average damage when a storm hits and the other half lose nothing. Figure H3 reports the results for the coastal group and the country as a whole. We aggregate counties using capital and consumption weights from the 2°C steady state.

The cost of risk for island capitalists rises with dispersion, and the response is convex. Since every second-order object is linear in Σ_0^2, the effects are small at low dispersion and grow more quickly as Σ_0 increases. Up to $\Sigma_0 = 0.3$, the coastal risk premium rises from 0.3 to 0.4 p.p. per year, and the lifetime consumption loss of coastal island capitalists rises from 3.5 to 5.5%. At the national level, the premium stays near 0.1 p.p. per year, while the capitalist welfare loss rises from 1.0 to 1.7%. Workers, in contrast, gain slightly. Coastal worker welfare rises from -0.02 to $+0.1$%, and coastal capital from -2.0 to -0.5%. As Σ_0 rises further to 1, the coastal premium reaches 1.2 p.p. per year and the coastal welfare loss 28%. The national premium and welfare loss rise to 0.2 p.p. per year and 9.6%, while coastal worker welfare and capital reach $+1.1$% and $+19$%.

The mechanism behind these patterns is precautionary investment. Since island capitalists have no other asset, greater risk leads them to hold more of their own capital as a buffer. Capital and wages rise, benefiting workers, while capitalists' welfare falls.

Large dispersion, however, reduces the accuracy of the second-order approximation. This approximation treats the additional own risk as a small correction around the deterministic steady state. The results at high Σ_0 should therefore be interpreted with caution.

Figure H3: Climate risk with dispersed storm damages within counties (island model).



Note: Island model, county level (3,037 counties). Stochastic steady state with the local and aggregate risk of a 2°C warmer climate, relative to the deterministic steady state of that climate, as a function of the within-county dispersion Σ_0 of storm damages across owners (Appendix H.2.1). Solid red line: coastal counties (coastal share of at least one half). Dashed grey line: country aggregate. Panel (a): capital risk premium, in percent per year, capital-weighted. Panels (b) and (c): lifetime consumption-equivalent welfare of island capitalists, weighted by consumption, and of workers, weighted by consumption and population, in percent. Panel (d): capital stock, in percent of the group's stock.

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